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THE MATHEMATICS TEACHER

VOLUME XVIII FEBRUARY, 1925 NUMBER 2

A STUDY OF THE FACTORS OF SUCCESS IN FIRST YEAR ALGEBRA

By EDWIN W. SCHREIBER
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CHAPTER I

INTRODUCTION

The problem of this investigation is to study some of the factors of success in first year algebra. The method of the investigation consisted chiefly of the administration of standardized tests which were given to 160 pupils in eight different classes of first year algebra in the Mathematics Department of the Proviso Township High School, Maywood, Ill. The tests were given at the time of the regular class periods during the last two weeks of school, May 27 to June 4, 1924. Of the 160 tested 63 (39%) were girls and 97 (69%) were boys. The following tests were given:

A. Courtis Research Test in Addition, Series B, Form 2. This test was given to determine the ability to add. It is composed of 24 exercises each consisting of a column of nine numbers of three digits.

B. Courtis Research Test in Multiplication, Series B, Form 2. This test was given to determine the ability to multiply. It is composed of 24 exercises each consisting of the indicated product of a four digit number with a two or three digit number.

C. Hotz First Year Algebra Scale in Equation and Formula, Series B. This test was given to determine the ability to solve equations and formulæ in algebra after a course of nine months in the subject. The test consists of 25 problems ranging in difficulty from very simple linear equations to equations containing radicals.

D. Hotz First Year Algebra Scale in Problems, Series B. This test was given to determine the ability to derive equations. It consists of 14 problems ranging in difficulty from very

simple relationships as: "If one coat cost x dollars, how much will three coats cost?"; to rather complex relationships as: "An open box is made from a square piece of tin by cutting out a 5-inch square from each corner and turning up the sides. How large is the original square, if the box contains 180 cubic inches?"

E. Otis Self-Administering Test of Mental Ability, Higher Examination, Form A. This test was given to obtain a measure of general intelligence. It is composed of 75 questions of a wide range.

F. The ability to succeed in first year algebra was measured by the semester mark received in the course. This is the record that is kept on file in the office and is used in all future transactions where algebraic ability is considered.

It will be the purpose of this study to consider carefully the scores made by these 160 first year algebra pupils in each of the above mentioned standardized tests and to interpret the findings in the light of what part they play as contributing factors to success in first year algebra. Chapter II will deal with an examination of the abilities. In the tests the number of "rights" is used as the basis for the statistical calculations. Chapter III will deal with the relationship of abilities as measured by correlation co-efficients of the zero order and the first order; it will also deal in a limited way with regression lines and equations. Chapter IV will give a detailed analysis of problems solved and failed in algebra as revealed by the tests. It will also deal with types of errors made in solving equations and formulæ and in deriving equations. Chapter V will give an analysis of failure and will deal with the abilities and intelligence of the 27 who failed to get their credit in the course in first year algebra. Chapter VI will give a summary of the findings and conclusions.

Chapter II

An Examination of Abilities

Arithmetic still holds, in public estimation, a prominent place as one of the fundamentals of both a liberal and vocational education. Nearly one-fifth of the total time spent in the classroom in the elementary school is devoted to arithmetic. Approximately one-fifth of the first year high school pupil's time

in the classroom is spent on algebra. This chapter is concerned with the actual status of certain arithmetic and algebraic abilities of first year algebra pupils. How well do they add and multiply in arithmetic and how well do they solve equations and formulæ in algebra? How well do they derive equations in algebra? The answers to these questions are of great value to the instructor of algebra as a guide to future instruction and to the administrator in determining curriculum policies.

Are children taking algebra above the average in intelligence? What is the general status of their intelligence as measured by a standardized test? Furthermore, are pupils studying algebra 15 years of age or is there a great range in age? And finally, how do first year algebra pupils vary in ability as measured by semester marks in the course?

Arithmetic skills are many and varying in complexity. They range from those of addition and multiplication to such complex products as ability to reason in certain situations. Measurement of the ability to add and multiply, the two most used mechanical abilities in the affairs of life as well as in future courses in the classroom, is comparatively easy and the tests have been thoroughly standardized through the excellent work of Mr. S. A. Courtis of Detroit, Mich.

A. Distribution of Abilities to Add.

Table 1 gives the scores in addition made by the 160 pupils in eight first year algebra classes at Proviso as measured by the Courtis Research Test in Addition, Series B, Form 2. In the table horizontal distance represents rate (number of examples attempted) and vertical distance represents accuracy. It shows very strikingly the great range in both rate and accuracy of pupils who have been exposed to supposedly the same training in the elementary school. For example, examine a rate of 11 examples attempted and note the range of accuracy. One pupil who attempted 11 got them all right; 4 had an accuracy of 90-99%; 4 had an accuracy of 80-89%; 5 had an accuracy of 70-79%; 3 had an accuracy of 60-69%; 2 had an accuracy of 50-59%; and 4 had an accuracy of less than 50%. In other words these 23 pupils had the same rate (11 examples attempted), but in accuracy they varied from less than 50% to 100%. Again, examine a certain level of accuracy, 70-79%.

TABLE 1
Courtis Test in Addition, Series B, Form 2
Number of Examples Attempted

	%	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Total	%
Accuracy	100						2		1	1	1		1	1		1					1						9	5.6
	90											2	4	2	1		1		2	2			1			1	16	10.0
	80							3	3	3	2	3	4	4	2	1	2	1		1			1				30	18.7
	70					2			5	3	2	4	5	3	1	3	2	1	1			1					33	20.6
	60						1	1		2	3	5	3	5	2	4	2				1	1					30	18.7
	50					1		2	1	1	1	4	2		1		1										14	8.8
	0-49					1	2	2	7	4	4	1	4	2								1					28	17.5
Total						4	5	8	17	14	13	19	23	17	7	9	8	2	3	3	2	3	2	0	0	1	160	

Median Scores: Rate 11.0 Accuracy 72.4% Rights 7.8

TABLE 3
Courtis Test in Multiplication, Series B, Form 2
Number of Examples Attempted

	%	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Total	%
Accuracy	100					1		2	5		2	1	2	2		1											16	10.0
	90											3	4	1	3		1		1			2					15	9.4
	80						2	2	2	2	4	5	6	4	1	1	2	1					2				34	21.2
	70								2	2	5	5	5	7	5	2			1		1						35	21.9
	60							5		2	3	3	1	2	2	1	1	4	1								25	15.6
	50			1				1		1	2		3	3	1												12	7.5
	0-49				1	1	1		2	2	4	3	5	1	2						1						23	14.4
Total				1	1	2	3	10	11	9	20	20	26	20	14	5	4	5	3	0	2	2	2				160	

Median Scores: Rate 11.1 Accuracy 75.7% Rights 8.1

A total of 33 pupils had approximately the same accuracy but varied in rate from 4 examples attempted to 20 examples attempted. In other words, accuracy remaining constant some pupils worked five times as fast as others. Or, looking at the results from the standpoint of greatest extremes, while one pupil attempted only 4 examples and got 1 right, another pupil attempted 24 examples and got 23 right. Thus the ratio in this case is 1 to 23 and yet both pupils are studying algebra with supposedly the same preliminary training in addition, and surprising as it may be to the inexperienced investigator, they both passed in algebra, the former with a mark of 80 and the latter with a mark of 90.

Table 1 also gives the median scores for rate, accuracy, and rights. Thus one half of the pupils attempted 11.0 (or more) examples, getting 7.8 (or more) right, with an accuracy of 72.4% (or more). The question naturally arises, how do these pupils compare in rate, accuracy, and rights, with the Courtis

TABLE 2

Comparative Data in Addition, Courtis Test, Series B, Grade 8

Comparative Standards	Rate	Accuracy	Rights
Courtis Standards	11.6	76	8.8
Boston, 1915	13.7	78	10.7
Small Cities, 1916	10.2	74	7.6
Pennsylvania Rural Schools	7.7	52	4.0
California Rural Schools	8.8	49	4.3
Gary, Ind., 1916	8.4	57	4.8
PROVISO, 1924	11.0	72	7.8

standards, and with other groups in different schools? Table 2 on this page gives this information. It should always be remembered that while tests reveal differences in achievement, from school to school, they do not in any way reveal the causes of the differences.

Figure 1 gives the frequency distribution for rights in addition and shows that 134 (84%) of the pupils get between 2 and 11 examples right. Figure 2 gives the percentage distribution for rights in addition. As measured by this test, 4 (2.50%) pupils have practically no ability to add. However, 2 of these 4 passed in algebra, one with a mark of 75 and the other with a mark of 80. As a group the ability to add has not been very

highly developed. Adults in various occupations work with a rate of about 14 examples attempted with an accuracy of about 80%.

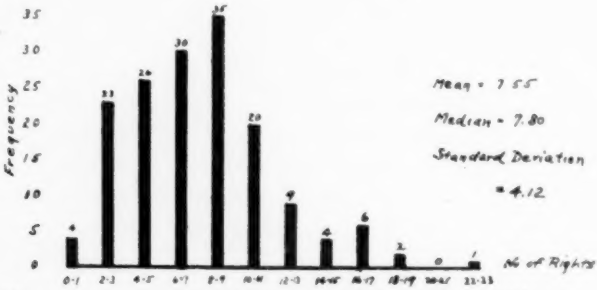

Fig. 1. Frequency Distribution of 160 Pupils in Addition.

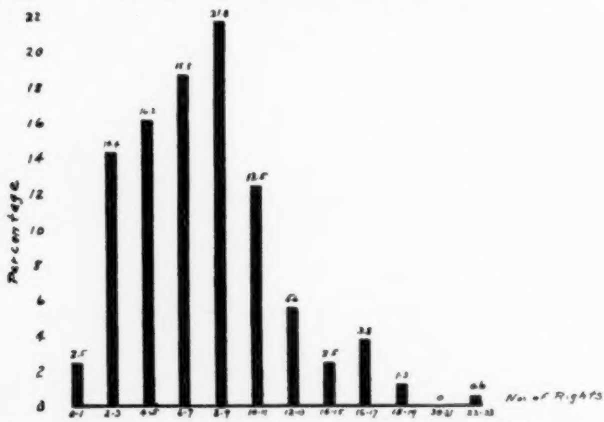

Fig. 2. Percentage Distribution of 160 Pupils in Addition.

B. Distribution of Abilities to Multiply.

Table 3 gives the scores in multiplication of 160 pupils at Proviso as measured by the Courtis Research Test in Multiplication, Series B, Form 2. Again there is a wide range among pupils in both rate and accuracy. Consider the 26 pupils having a rate level of 11 examples attempted their accuracy varies from less than 50% to 100%. Or take a constant accuracy level of 80-89%; 34 pupils at this accuracy level vary in rate from 5 examples attempted to 21 examples attempted. In short, some pupils work more than four times as fast as others in multiplication with the same degree of accuracy. Further-

more, considering the greatest extremes, one pupil attempted only 2 examples in multiplication getting 1 right, while two other pupils attempted 20 examples getting 19 right. All three passed in algebra, the former with a mark of 75 and the two latter with marks of 85 and 95.

The median scores at the bottom of Table 3 gives the following: median rate 11.1, accuracy 75.7%, rights 8.1.

How do these results compare with Courtis standards and with other schools? Table 4 answers this question.

TABLE 4.
Comparative Data in Multiplication, Courtis Test, Series B, Grade 8

Comparative Standards	Rate	Accuracy	Rights
Courtis Standards	11.5	81	9.3
Boston, 1915	11.6	83	9.6
Small Cities, 1916	11.0	81	8.9
20 Cities, Ind., 1914	10.2	71	7.2
Gary, Ind., 1916	8.4	67	5.6
PROVISO, 1924	11.1	76	8.1

Figure 3 gives the frequency distribution of rights in multiplication of the 160 pupils at Proviso. More than three-fourths of the pupils (77.5%) get between four and eleven examples right. Only eight do very well in multiplication. Seven pupils (4.4%) show practically no ability in multiplication as measured by this test. Of these seven only two failed in algebra, three received a mark of 75 and two a mark of 85.

TABLE 5
Hotz Scale in Equation and Formula, Series B
Number of Problems Attempted

	%	13	14	15	16	17	18	19	20	21	22	23	24	25	Total	%
Accuracy	100															
	90										3	1	4		8	5.0
	80					1				2	5	2	9	4	23	14.4
	70									2	2	4	12	6	26	16.2
	60								3	1	5	7	10	7	33	20.6
	50		1							1	7	5	11	4	29	18,1
	0-49	1	1				2		1	7	4	7	15	3	41	25.6
Total		1	2			1	2		4	13	26	26	61	24	160	

Median Scores: Rate 24.0 Accuracy 63.0% Rights 15.2

C. Distribution of Abilities to Solve Equations and Formulae.

Table 5 gives the results of the 160 pupils at Proviso in solving equations and formulæ as measured by the Hotz Test, Series B. The median rate of 24.0 shows that the time allowed (40

minutes) was sufficient for most pupils. There are six stragglers but, of these, four failed the course and the other two barely passed with a mark of 75. Because of the high rate of attempts (24) naturally there would be many errors, and many

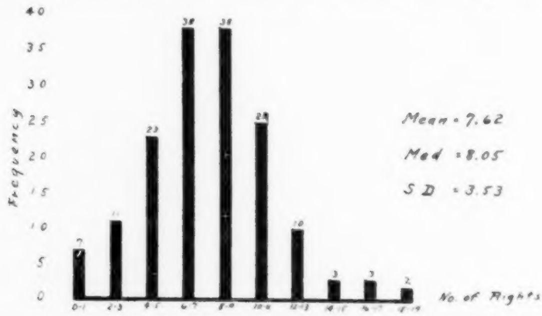

Fig. 3. Frequency Distribution of 160 Pupils in Multiplication.

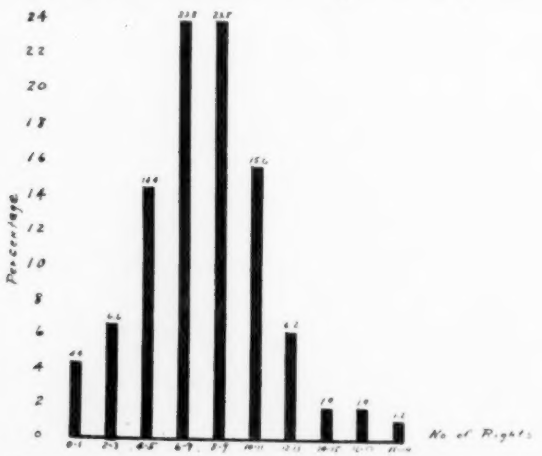

Fig. 4. Percentage Distribution of 160 Pupils in Multiplication.

wrong methods of procedure, bringing the median accuracy down to 63.0%, or 15.2 rights. About one-fourth (25.6%) of the pupils had an accuracy of less than 50%.

Mr. Hotz in his "Teacher's Manual for First Year Algebra Scales," 1922, gives only median scores for rights as a basis for comparison. Table 6, on this page, gives his standards and also those of other cities. The group at Proviso compares favorably with the achievement of pupils in other schools.

TABLE 6
Comparative Data in Hotz Equation and Formula Scale
and Problem Scale, Series B

Comparative Standard	Equation and Formula	Problem
Hotz Standard	16.0	7.5
Wisconsin Cities, 1918	17.2	8.2
Wellington, Kans., 1920	16.7	9.4
Elizabeth City, N. C.	12.8	8.1
Andover, Mass.	12.8	---
PROVISO, 1924	15.2	6.2
PROVISO, (Without Failures)	16.3	7.3

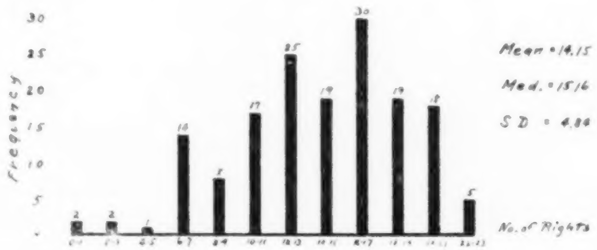

Fig. 5. Frequency Distribution of 160 Pupils
in Hotz Equation and Formula Scale, Series B.

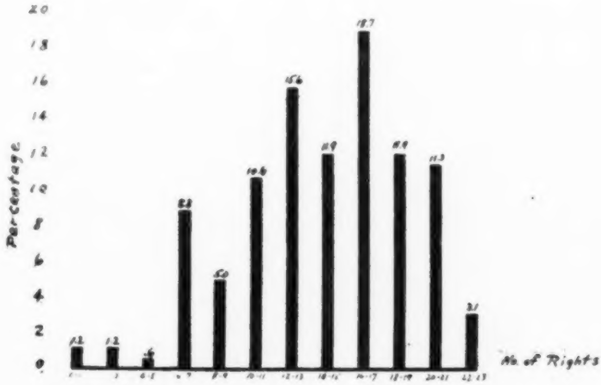

Fig. 6. Percentage Distribution of 160 Pupils
in Hotz Equation and Formula Scale, Series B.

Figure 5 is the frequency distribution and Figure 6 the percentage distribution of the 160 pupils at Proviso in Hotz Equation and Formula Scale, Series B. The variability of the group is large, as the standard deviation of 4.84 indicates. Hotz

standard deviation in his original study was 4.1. All five pupils who made a score of five or less failed the course in algebra and seven of the 14 who made a score of six or seven failed the course in algebra.

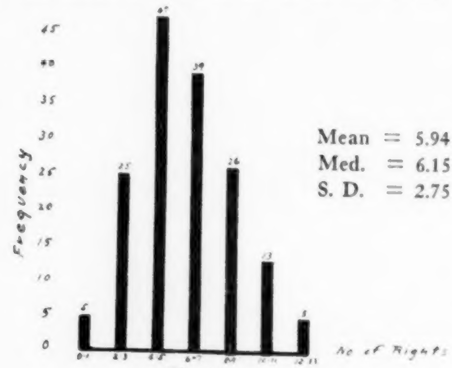

Fig. 7.

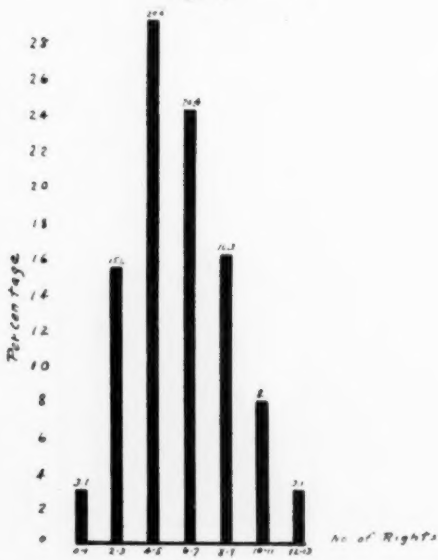

Fig. 8.

Figs. 7 and 8. Frequency and Percentage Distributions of 160 Pupils in Holz Problem Scale, Series B.

D. Distribution of Abilities to Derive Equations.

The teaching of algebra has two distinct divisions, namely: (a) the machinery of algebra, (b) and the problem or reason-

ing material in the working of which the machinery is used. How well do the 160 pupils at Proviso use this algebraic machinery in deriving equations? Table 7 tells the story. Most of the pupils are ready to set up some kind of machinery and run it at maximum speed (14) but the crash follows all too soon as shown by an accuracy of only 47.1%. Figures 7 and 8 reveal the fact that more than one-fourth (29.4%) of the group get only four or five problems right, and nearly one-half (48.1%) get five or less right. Of the five getting only zero or one right four failed in algebra and one passed with a mark of 75. But

TABLE 7

Hotz Scale in Problems, Series B
Number of Problems Attempted

	%	6	7	8	9	10	11	12	13	14	Total	%
Accuracy	100		1			1					2	1.3
	90					1	1	1	2		5	3.1
	80			1	2	3	2		1	3	12	7.5
	70		1	1		2	1	1		9	15	9.4
	60			2	1	1	1	1	1	14	21	13.1
	50	1		3	1		2	2	4	7	20	12.5
	0-49			2		6	4	6	11	56	85	53.1
	Total	1	2	9	4	14	11	11	19	89	160	

Median Scores: Rate 14.0 Accuracy 47.1% Rights 6.2

of 25 getting two or three right only 10 failed while nine passed with a mark of 75, three with 80, two with 85, and one with 90. This indicates that the test was too difficult. Furthermore, only 18 (11.2%) could do more than ten problems correctly, additional evidence that the test was too difficult. However, the group as a whole is poor in ability to derive equations as measured by this test, revealed by a comparison with the Hotz standard and those of other schools shown in Table 6.

E. Distribution of General Intelligence.

The distribution of general intelligence as measured by the Otis Higher Examination, Form A, is given in Figures 9 and 10. An IQ of 100 represents normal intelligence. The median for the group is 101.3, showing that pupils who take the course in algebra are on the average no brighter than other children. Of the six who have the lowest IQ, three failed the course in algebra,

two barely passed with marks of 75, and one passed with a mark of 80. The last mentioned case is one of a girl nearly 20 years old who is of junior rank in the high school. Her maturity no doubt was the factor that helped her to stick to the task while

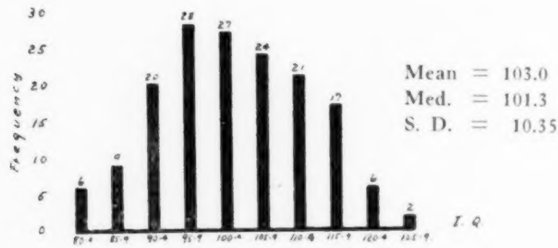

Fig. 9. Frequency Distribution of IQs as Determined by Otis Higher Examination, Form A.

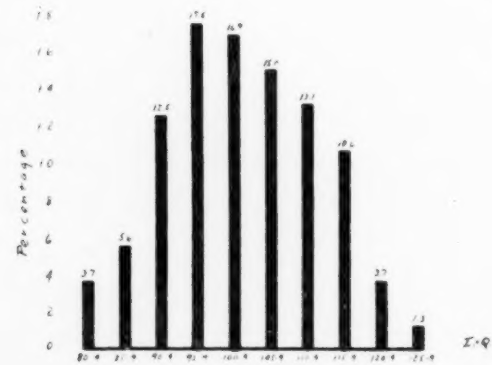

Fig. 10. Percentage Distribution of IQs as Determined by Otis Higher Examination, Form A.

others gave up. One of the two showing the highest IQ received a mark of 95 in algebra while the other only received a mark of 80. However, in general, those who failed in algebra had a low IQ, while those who succeeded very well had a high IQ.

Figures 11 and 12 give age distributions and reveal the rather startling fact of the wide range in age of pupils studying first year algebra. Of course a few of the older ones are upper class-

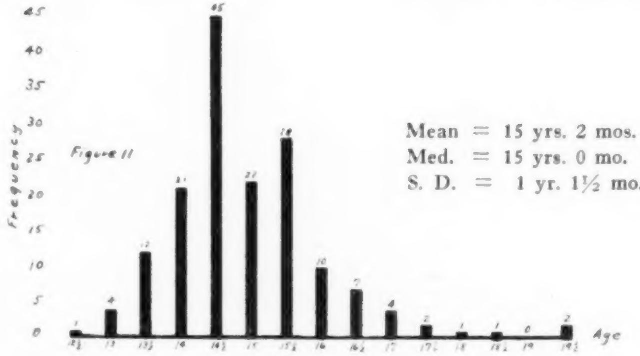

Fig. 11.

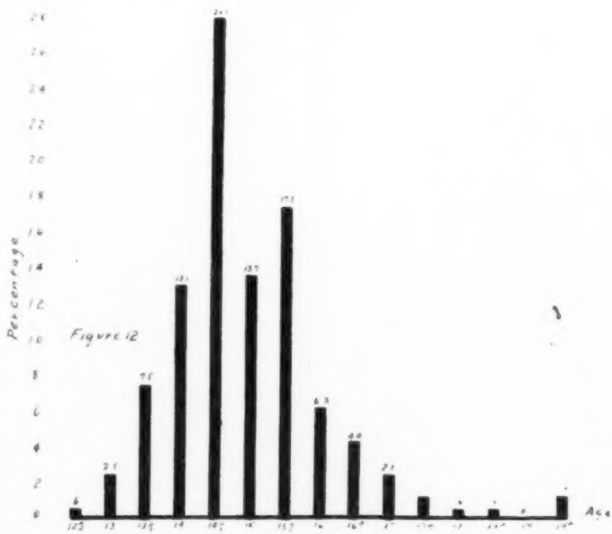

Fig. 12.

Figs. 11 and 12. Frequency and Percentage Distributions of 160 Pupils in Age.

men who decided to go to college and need algebra as an entrance unit. The variability, however, is not very large as nearly two-thirds of the group are between 14 and 16 years of age.

F. Distribution of School Marks in Algebra.

Figures 13 and 14 give the distribution of semester marks in algebra. These marks are kept on file in the high school office. Twenty-seven (16.8%) of the pupils failed in the course in alge-

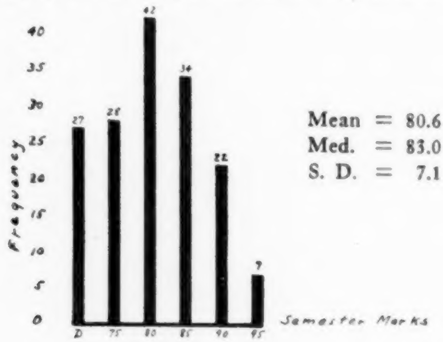

Fig. 13.

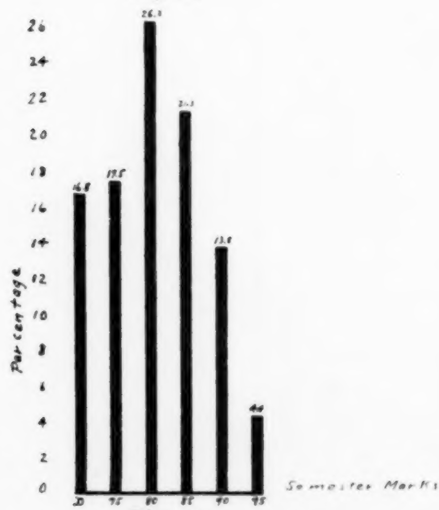

Fig. 14.

Figs. 13 and 14. Frequency and Percentage Distributions in Semester Marks in Algebra.

bra. The "D" stands for deficient and means failure in the marking system at Proviso. Of those who passed the course in algebra 28 received a mark of 75, 42 a mark of 80, 34 a mark of 85, 22 a mark of 90, and seven a mark of 95.

To be continued

STUDENT DIFFICULTIES IN EXERCISES IN GEOMETRY

By WINONA M. PERRY
The Lincoln School

What are the difficulties which confront a student when he is attacking an exercise in geometry? Are they, in reality, better described as just one difficulty—that of not being able to "see" or "do" a problem? Or are they reducible to more specific difficulties, which can be considered more or less separately? If so, could groups of exercises be planned which will indicate the improvement of individuals in their ability to solve increasingly difficult exercises?

If we face the problem from the point of view of geometry, the following is a possible analysis:

1. Prove one conclusion (e. g., that triangles are equal).
2. Prove one derived conclusion (e. g., corresponding parts in equal triangles).
3. Prove one derived conclusion with overlapping figures.
4. Prove two derived conclusions from one pair of equal triangles.
5. Prove two pairs of triangles equal.
6. Prove derived conclusions from two pairs of equal triangles.

The writer felt that it was possible to define more clearly these difficulties if they were studied from the angle of the students' responses. Accordingly, exercises were analyzed into the number of (1) essential or crucial steps, (2) possible favorable responses, (3) rejected or "blocked" responses, (4) derived conclusions, (5) elements, (6) relations, as well as, (7) relations complicated by the presence of overlapping figures and construction lines. From this analysis, a list of ten exercises was chosen and arranged in the following form:

SUGGESTED RATING FOR DIFFICULTY

I. Are the following originals arranged in order of the probable difficulty experienced by students in the solution of these originals?

Underline Yes or No.

II. If not, please state what rearrangement you would suggest, and why.

1. If a diagonal of a quadrilateral bisects those angles whose vertices it joins, the diagonal divides the figure into two congruent triangles.

2. The line joining the vertex of an isosceles triangle to the midpoint of the base bisects the angle at the vertex.

3. Given the isosceles triangle ABC with BC the base; also D and E points on BC equally distant from B and C respectively. To prove $AD = AE$ and $\angle BAD = \angle CAE$.

4. If the base of an isosceles triangle is divided into three equal parts, the line segments joining the points of division to the vertex are equal.

5. A point on the bisector of an angle at the vertex of an isosceles triangle is equally distant from the extremities of the base.

6. Perpendiculars drawn to the equal sides of an isosceles triangle from their midpoints and terminating in the base or the base produced are equal.

7. Bisectors of the base angles of an isosceles triangle and terminated by the opposite sides of the triangle, are equal.

8. If the bisector of an angle of a triangle is perpendicular to the opposite sides, the triangle is isosceles.

9. Two triangles are congruent if two sides and the median (line segment joining the vertex and the midpoint of the opposite side) to one of these sides are equal respectively to two sides and the corresponding median to the other.

10. If a diagonal of a quadrilateral bisects both angles whose vertices it joins, the two diagonals of the quadrilateral are perpendicular to each other.

These exercises were sent in March, 1924, to eighty-two teachers and students enrolled in classes (the Teaching of Mathematics and the Teaching of Geometry) in Teachers College. Replies were received from fifty-seven, although only fifty-four (65.8%) could be tabulated. Furthermore, these ten exercises were printed in groups of two on a sheet, and solved by twenty-eight students in The Lincoln School (both boys and girls), by thirty-three students in Stuyvesant High School (boys), and by thirty-three students in the Wadleigh High School (girls)—each in New York City. As a check upon the rankings of the teachers, the same exercises were rated in October, 1924, by members of a class in the Teaching of Mathematics in Teachers College; but the procedure differed, in that now these ten exercises were placed on ten separate sheets of paper and put in an envelope in random order.

The consensus of opinion, including the ranking of each teacher for each of the ten exercises, was found by averaging—thus assigning credits of difficulty for each exercise. Also, the order

of difficulty, as determined from the actual solutions of ninety-four students, was based upon the percentage of correct solutions per exercise. These two methods result in the following orders:

ORDERS

Given	Teachers Mar., 1924	Teachers Oct., 1924	Students	% Correct	% Completing the Exercises
1	2	1	2	92	98.9
2	1	4	3	90	98.9
3	3	3	4	84	94.6
4	4	2	5	70	94.6
5	5	6	1	68	98.9
6	8	5	7	63	74.4
7	7	8	8	52	61.7
8	6	7	6	46	84.0
9	9	9	9	18	28.7
10	10	10	10	9	17.0

Columns 2 and 4 indicate that Ex. 1 was more difficult, and Ex. 8 less difficult, than the positions assigned to them in the given order.

On the basis of these orders, of the reasons stated for rearrangement of the given order, and of the analysis of reasoning by Dr. E. L. Thorndike in *Educational Psychology*, Vol. II, there results this experimental outline of student difficulties in the solution of exercises in geometry:

STUDENT DIFFICULTIES IN EXERCISES

A. Linguistic: statement, elements, relation between elements.
B. Analysis of facts into their elements.
  I. Figure (basis of reasoning).
    1. Complexity, due to "potency of words and phrases."[1]
      1. As perpendiculars, bisectors, medians, producing sides.
      2. As *types* of figures.
    2. Unfamiliarity, due to "inadequacy of connections."[1]
      1. Relations between elements.
        i. Number.
        ii. Obscuring of relations, as { overlapping figures. / construction lines.
      2. Number of non-available, habitual responses—as diagonal connecting *two* bisected angles; i. e., less recognition of partial identity of certain elements in the situation.

[1] Thorndike, E. L. "Psychology of Thinking," *Psychological Review*, 1917.

II. Reasoning—activity.
   1. Selection.
      1. Rejected responses (neglecting relations or aspects of the figure).
      2. Accepted responses (from hypothesis, from possible conclusions).
   2. Use in right relations.
      1. Number of essential steps—"purposive thinking."
      2. Detection of the crucial step, or steps.
   3. Drawing conclusions.
   4. Judging conclusions—leading to bonds being strengthened, from satisfaction.

The last problem, which was mentioned above, is even more in the experimental stage than that of determining the difficulties as students confront the exercises. The present writer has planned six tests on the basis of the above outline, in an attempt to study the improvement shown by students in their solutions of exercises in Book I; the results, however, are yet to be determined.

SOME TRUE-FALSE EXAMINATIONS FOR USE IN GENERAL MATHEMATICS

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Many influences have led in recent years to an intensive study of the content and of the administration of secondary school mathematics.[1] One administrative phase studied in particular has been the testing and examination of pupils. Tests and examinations have been used first, for ability, second, for the purpose of securing marks to send to parents or educational institutions, and third for purposes of diagnosis. That examinations are of wide interest and consequently have attained most serious attention in many quarters is evidenced by the fact that one examining board of national scope[2] has an annual budget of $170,000. That any type of examination, then, should remain constant is hardly to be expected. Efforts to secure new forms are being made and the tests are referred to ordinarily as "New Type" examinations. The chief claims for these new type examinations are that they save time, have a wide scope, and stimulate better relations between the examined and the examiner. It is also hoped that such tests will have diagnostic properties of importance to the pupil and to the teacher.

This new type or less formal type of examination has been discussed by McCall.[3] A phase of these newer types of examination forms that has caused considerable discussion and expression of conflicting opinions is the scoring of these brief answer tests. One of the earliest of these criticisms is found in Dean Hahn's article.[4] To summarize his objections he has made and reported an experiment to show that the guessing factor is a danger in these tests of alternate responses. He finds that the final score represents an unknown quantity, and that such tests ought not to be the basis for determining mental age, for vocational purposes, for classifying children for school work, for studying in-

[1] See Bibliography at the end of this article.

[2] College Entrance Examination Board.

[3] "A New Kind of School Examination," *Journal of Educational Research,* January, 1920.

[4] Hahn, H. H.: "A Criticism of Tests Requiring Alternative Responses," *Journal of Educational Research,* Vol. VI, 1922, pp. 236-240.

dividual differences, nor for diagnostic and remedial work. He further states that their influence is reactionary and that they are a form of question condemned by pedagogy, while their principal reason for being is to avoid trouble in answering questions and scoring answers.

One of several replies to Dean Hahn's findings is that of Mr. Barthelmess.[1] This reply takes a criterion, correlation. He says the "true-false" has a higher correlation than the conventional examination. It was found at Columbia University, New York, that the newer type of examination yields a coefficient of .90 (several types were used, true-false being one). True-false alone had a reliability of .80. Toops[2] found that of recall, recognition, and true-false, the last yielded results as reliable as the others for equal amount of examination time given. The power of the true-false to cover in a given time far more ground than the ordinary test can, adds more to the relative reliability than any such inaccuracies (guessing) can subtract. The ordinary test suffers from the fact that the smaller number of questions, the greater the effect of chance vitiations. That alternate-response tests may be so made that they constitute a test of judgment and thought organization has been demonstrated by Wood. It is true that any test inviting dichotomous responses is weak as an individual diagnosis, although useful for general diagnosis.

A second reply to Dean Hahn is by Mr. Odell.[3] Mr. Odell takes exception in several respects. He says an advantage of these tests is that they measure the subject that it is desired to measure, and that a test that yields an objective score is preferable on that gound alone to almost anything upon which the scores have no meaning.

The debate over scoring the true-false tests is continued by Paul V. West.[4] "This method of scoring should be further investigated from various points of view. It is clear that it is of very doubtful reliability for group testing and especially so

[1] Barthelmess, H. M.: "Reply to a Criticism of Tests Requiring Alternative Responses," *Journal of Educational Research*, Vol. 6, November, 1922; pp. 357-359.

[2] Toop of Ohio State, quoted by Barthelmess.

[3] Odell, C. W.: "Another Criticism of Tests Requiring Alternative Responses," *Journal of Educational Research*, April, 1923; pp. 326-330.

[4] "A Critical Study of the Right-Minus-Wrong Methods," *Journal of Educational Psychology*, June 8, 1923; pp. 1-9.

for analysis of individual ability. . . . Whatever doubts may eventually be held regarding the reliability of this method must carry with them in related degree doubts regarding the reliability of similar methods of grading tests which use three or more alternatives."

In a field closely allied to the one of mathematics, physics, our new type (true-false) of examination has found favor in Columbia College and has been reported by Mr. H. W. Farwell as some new type tests[1] in Columbia College, in general physics courses, during 1922-23. There were used true-false, completion, recognition, and association tests. The true-false was used more than any other. It was used in the final examination, which included 100 statements, and occupied one-third of a three hour period. The entire physics department contributed statements. This was the case in June, 1922, and June, 1923. The other two hours of the period were given to problem work. The department's decision as to marking the true-false statements was to use + 1 for correctly marked statements; — 1 for incorrectly marked one, and 0 for an unmarked one.

Perhaps one of the latest influential steps to be taken is that of the College Entrance Examination Board.[2] The Board, in its April 5, 1924, meeting, unanimously passed the following resolution: "Whereas, It seems to the Board desirable to make a practical test of the value of the new type examination. Resolved, That the examiners be required to modify the present type of examination in elementary algebra (Algebra A_1, A_2, A) and in ancient history so that the papers in each of these divisions shall include questions of the new type, and that examinations be set as soon after April, 1924, as possible." But while the Board has some new type examinations prepared at their suggestion by the Institute of Education Research, Teachers College, Columbia University, the resolution was adopted too late to become effective this year. An examination of these questions shows the ones for algebra to be of the same nature as the Thorndike Algebra Test.[3] There are forty questions that

[1] "The New Type Examination in Physics," *School and Society*. Vol. 19, March 15, 1924; pp. 315-322.

[2] *School and Society*, April 19, 1924; pp. 466-468.

[3] "The Reorganization of Mathematics in Secondary Education," the National Committee on Mathematical Requirements, 1923; pp. 358-361, particularly the 39th on p. 361, a four part true-false question.

are of the pure alternative answers type. One other, the 29th in the series, is of the completion type, giving three possible ways of completing the unfinished statements based on a formula with stated changes in various items of the formula. The Board has similar series of algebra examinations "prepared for the information of teachers and to facilitate experiments and observations in secondary schools with a view to determining the value of such tests."

Leaving the historical side of the new type examination, and taking up the preparation of one of them, we find rather full directions from a recent report by W. S. Monroe.[1] Under "Directions for Constructing a True-False Examination," he says:

I. Prepare a list of statements covering in some detail the portion of the subject on which the pupil is to be examined. Some statements can be changed so that they are false. The untruth should not be too obvious. Statements selected should require an acquaintance with the subject.

II. The number of true statements should approximate the number of false ones, and the arrangement such that there is no regular sequence between true and false statements.

III. The list should be not less than 50.

IV. The examination should be mimeographed or printed so that each pupil has a copy. Answers may be given in the margins, or written on a separate sheet, if desired, using numbered blanks. A less desirable way of giving is reading the statement to the class. This does not afford sufficient opportunity to study the statements. Also the class may give hints of answers.

V. Give specific directions regarding answering exercises about which the pupils are uncertain. Either instruct the class to guess or not to guess. "First, go through the list quickly and mark all that you know for certain, then go back and study out the harder ones. Do not guess; the chances are against you on guessing. Don't endanger your score by gambling on those questions about which you know nothing."

Scoring. The score equals the number of statements answered correctly minus the number answered incorrectly. Exercises not attempted are not counted.

Usefulness. 1. To test the acquaintance of a class with a wide range of facts.

2. Appropriate for use at the end of teaching unit.
3. Simple in administration.
4. Scores can be translated into grades.
5. Avoids use of judgment in scoring.
6. Scoring highly objective.
7. Examination may be comprehensive, much more so than traditional examination.
8. Time saved in giving and scoring.
9. Pupils may do scoring.
10. Discussion of mooted points may be had "on the spot."

[1] "The Present Status of Written Examinations and Suggestions for Their Improvement," University of Illinois, Bulletin 17, November 2*6, 1923.

Another favorable position regarding true-false tests is taken by Dr. William A. McCall.[1] He claims these advantages for this particular type of new examination:

1. The examiner may cover a wider range of ability per unit of time. 2. Use of the type is likely to improve the relation between teacher and pupils. 3. The type is more enjoyed by the pupils. It offers an opportunity for a contest where the rules are fair and offers them a chance for a large participation in the examination. 4. It is more enjoyed by the teacher, for the scoring is easy, rapid, and automatic when he does the scoring, and far more rapid when the pupils do the scoring. 5. It is more educative for the pupils. 6. It gives the teacher a fuller knowledge of the conditions. Testing is likely to become more frequent, and this means more complete and timely information about the abilities and difficulties of the various pupils, and about the successes and failures of teaching efforts. 7. It is a genuine honesty test, and shows the beginnings of a technique for measuring in satisfactory fashion this valuable character trait. "Most of these claims rest upon logical probability and a limited experience and not upon experimental data. This last is needed and will follow in time."

Teachers accustomed to basing their customary tests upon problem material or theorems, may continue to give these problems or theorems as the major part of the test and use a new type test for the minor part. At Columbia College[2] for several years, the physic department has based the final examination upon problems for two-thirds of the time and a true-false set of statements for one-third of the time.

This seems an excellent plan for gradually getting acquainted with the proposed new tests. To suggest a few departures from tractional tests, I made use of a general mathematics text[3] on a basis for three attempts at something a bit different from the kind of question found in the Board examinations.

My first effort was to use figures accompanied by data as a basis for a conclusion. In the second case I expect the pupil to draw a figure, if he chooses, and then make his conclusion. For the two sets of true-false statements, 82 and 32 in number, respectively, I followed the text of the first two chapters of the Reeve book.[3] Chapter one contains propodutic matter, largely, and chapter two, the customary congruency theorems. My chief objective is to find something that may lead to using tests for diagnostic purposes and thus to improve teaching results in

[1] *How to Measure in Education.* Macmillan Company, New York, 1922; pp. 128-133.

[2] Farwell, H. W.: *The New Type Examination in Physics, School and Society.* Vol. XIX, March 15, 1924; pp. 315-322.

[3] *General Mathematics,* Book II, Reeve, W. D. Ginn & Co., New York, 1922.

mathematics. Such use for mathematics teachers was urged at the National Council of Mathematics Teachers, Chicago, February 23, 1924.[1]

The nature of my first attempt is suggested by the following:

1. Given: $\triangle ABC$ and $\triangle ABC'$; with AB common and $BC = BC'$. Conclusion: $ABC \cong ABC'$.

If the conclusion is true, place a check in the square under "true." If false, place a check in the square under "false." (Figures omitted here.)

2. Notice that certain angles are known in this figure. Conclusion: $ABD \cong ABE$.

If the conclusion is true, place a check ($\surd$) in the square under "true." If false, place a check in the square under "false."

In $\triangle ABC$: $AC = BC$. Conclusion: AEB is equilateral.

If the conclusion is true, place a check ($\surd$) in the square under "true." If false, place a check in the square under "false."

3. A teacher asked a boy to make a drawing of the school baseball ground. This is the figure that he drew. Conclusion: the drawing is a scale drawing of the diamond.

If the conclusion is true, place a check ($\surd$) in the square under "true." If false, place a check in the square under "false."

The accompanying three statements concerning congruency require a student's close attention to judge correctly.

1. If the converse of the following theorem is true, place a check in the square beside "true." If false, place a check in the square beside "false."

If the opposite sides of a quadrilateral are parallel, they are equal.

2. If the converse of the following theorem is true, place a check in the square beside "true." If false, place a check in the square beside "false."

If the altitudes of a triangle are equal, the triangle is isosceles.

3. Check in the proper square, according as the converse of the theorem below is "true" or "false."

If equal oblique lines are drawn from a point on the perpendicular to a given line, the oblique lines form angles with the given line.

[1] Reeve, W. D. A Better Use of Tests in Mathematics; Mathematics Teacher, March, 1924.

The last of the three types, and the most extensive, should be duplicated somehow, with the left hand column of "true" or "false" left vacant for the examinee to supply:

"TRUE-FALSE TEST"

True 1. Arithmetic and algebra are kinds of number work.
False 2. In studying geometry we deal primarily with numbers.
False 3. Geometry deals only with points, lines, and surfaces.
False 4. A line has width.
True 5. There is a definite beginning and ending to a line segment.
True 6. Collinear points lie in the same straight line.
False 7. The line from Chicago to New York to New Orleans is a curved line.
False 8. Only 7 straight railroad lines can be drawn through a point on a map.
True 9. A straight line is the shortest distance between any two parts of Chicago.
True 10. The windshield of an automobile has a plane surface.
False 11. In geometry we study the color and the weight of cubes.
True 12. Solid geometry means spatial geometry.
True 13. In a rectilinear figure the lines are straight.
False 14. A tennis court has the shape of a curvilinear figure.
False 15. A circle is bounded by points.
False 16. Equiangular means many sided.
True 17. When a figure is regular it has equal angles and equal sides.
False 18. Compasses are tripods because they have three feet.
True 19. Lines may be measured with rulers or compasses.
False 20. A sheet of squared paper is just as long as it is wide.
False 21. All line segments have the same length.
True 22. Geometric surfaces are equal or unequal.
True 23. A line may have its length named 7 in. or a, or GK.
True 24. There is no limit to an angle of XOA.
False 25. The watch hands move counter clockwise.
False 26. One perigon equals four straight angles.
False 27. An acute angle is greater than an obtuse angle.
True 28. A right angle is less than an obtuse angle.
True 29. An acute angle is less than a right angle.
False 30. If A K H is an angle H is at the vertex.
False 31. There are more exterior angles to a triangle than there are interior angles.
True 32. The string around the outer surface of a new, blown up bicycle tire forms a circle.
True 33. Many radii can be drawn in a circle.
False 34. The circumference equals twice the diameter.
True 35. The radius equals one half the diameter.
True 36. Keep off the diameter and you keep off the center of a circle.
False 37. Noah's ark was part of a circle.
True 38. When two radii form an angle it is a central angle.

False 39. A semi-circle equals three quadrants.
False 40. Compasses measure angles.
True 41. The decree is the unit in angular measurement.
True 42. The scale on the protractor runs from 0° to 180°.
True 43. Angles may be measured inside or outside of doors.
False 44. Adjacent angles have nothing in common.
True 45. We add in arithmetic, in algebra, and in geometry.
False 46. Addition and subtraction of angles are performed with pencil and paper.
False 47. The larger the central angle the smaller the arc on the circle.
True 48. If two lines form a right angle either one is a perpendicular to the other.
True 49. A plumb line held at the mid point of a long floor crack, is the perpendicular bisector of the crack.
True 50. To divide a stick into two equal lengths is to bisect it.
False 51. In each triangle there may be drawn two medians to each side.
False 52. An altitude of a triangle is a line drawn from any vertex to the mid point of the opposite side.
True 53. Bisecting an angle of 37° gives two angles of 18½° each.
False 54. When a person's arms form equal acute angles with his body they are parallel.
False 55. Every parallogram is a rectangle.
True 56. If every side of a four-sided figure is equal to every other side and every angle is a right angle the figure is a square.
False 57. A rhombus has neither equal sides nor angles.
False 58. The diamond on a playing card is equiangular.
True 59. A runner running the bases in a ball game runs around a rhombus.
True 60. The sides of a triangle might be lettered 1_1, 2_2, 3_3.
False 61. If two angles have their sides parallel left to right and right to left, the angles are equal.
False 62. If $X + Y = 90°$, either X or Y is the supplement of the other.
True 63. When a line is the perpendicular bisector of a line the angles formed are supplementary.
True 64. Vertical angles are also acute angles or obtuse angles or right angles.
False 65. If two parallel lines are cut by a transversal all the angles formed are equal.
True 66. If two parallel lines are cut by a transversal there are four interior and four exterior angles formed.
True 67. If one of the angles formed by a transversal and a pair of parallel lines is 30° there will be four obtuse angles formed.
False 68. If two lines are cut by a third line the alternate exterior angles are equal.
True 69. The T-square is an instrument for drawing parallel lines.
False 70. A T-square has one square part.
False 71. In $\triangle ABC$, $\angle A + \angle C + \angle A = 2$ st $\angle$s.
True 72. The angles of a triangle might be 70°, 30°, and 80°.
True 73. The third angle of a triangle equals 180° minus the sum of the other two angles.

False 74. A triangle may have three acute angles, or right angles.

True 75. If one angle of a triangle is 70° one of the other angles may be obtuse.

False 76. An isosceles triangle is also equilateral.

True 77. If $\angle A = 25°$, $\angle B = 130°$, $\angle C = 25°$ the triangle ABC is isosceles.

True 78. If the sides of a triangle are 5, 8, and 6, it is a scalene triangle.

False 79. Quadrilaterals are always squares.

False 80. The sum of the interior angles of a square is more than the sum of the interior angles of a trapezoid.

False 81. A triangle may be constructed from any three of its parts.

True 82. Any part is less than the whole.

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REPETITION OF ERRORS IN ALGEBRA[1]

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We think in conventional ways. Even though we may know the scientific facts concerning a subject, we may yet think and even act about it in popular terms. Perhaps we are prone to do this with regard to errors in algebra. We believe that learning in habit formation. We know that errors are one form of response to a situation, and that responses may become habitual. But we seem inclined to act as if errors were isolated and could be corrected without affecting future right responses. We do not always pay sufficient attention to the fact that a wrong answer, an error, may be a part of a series in the formation of a wrong habit, just as a correct answer may be part of a series in the formation of a correct habit.

The following study was undertaken in order to follow up errors made in algebra to see whether isolated mistakes, having no apparent connection with each other, would be more common, or whether mistakes were more apt to be repeated—to see whether mistakes of different kinds would be made by the same pupil in answering the same problems, or whether mistakes would be repeated and so related to each other as to form links in a habit-forming chain.

In order to study this relationship between errors, a set of twelve very simple problems in algebraic addition, subtraction, multiplication, and division were given to a group of 9As and 9Bs in a junior high school. These twelve problems (marked with an asterisk in the following paper) were inserted among other problems and given to the classes one day toward the end of the semester.

Paper 1

Subtract:

-11	$2x$	$9bc$	$-7b$	$4c$
$-\ 3$	$5x$	$-7bc$	$3b$	$-6c$
* ______	______	______	* ______	* ______

[1] Read before the section on Education of the A. A. A. S., on January 1, 1925. *A Summary.*

Divide:

$\frac{10y}{20}$	* $\frac{-b^2}{-b^2}$	$\frac{2n}{4}$	* $\frac{9m^3}{-3m}$	* $\frac{-24x}{8}$

Multiply:

$2a$	$-9a$	$2y$	$5d$	$-3m^3n$
$-4b^2$	$3a^2$	$-7z$	$7x$	$-2mx$
*	*			*

Add:

$5a$	$4b$	$-8mn$	$-8a^3$	$7y$
$2a$	$-3b$	$11mn$	$-2a^3$	$3y$
*		*	*	

Then the same twelve problems (still marked with an asterisk in the following paper) were inserted in a different set of problems, and given to the classes on a second day.

Paper 2

Multiply:

$2a$	$-3m^3n$	$-9a$	$3b$	$-5s$
$-4b^2$	$-2mx$	$3a^2$	$7y$	$-2s$
*	*	*		

Divide:

* $\frac{-24x}{8}$	* $\frac{9m^3}{-3m}$	$\frac{6x^2}{3x}$	$\frac{4a}{5a}$	* $\frac{-b^2}{-b^2}$

Add:

$-8a^3$	$5a$	$-2y$	$-8mn$	$4b$
$-2a^3$	$2a$	$-7y$	$11mn$	$9b$
*	*		*	

Subtract:

$4c$	$-7b$	-11	$7a$	$6bx$
$-6c$	$3b$	-3	$8a$	$-4bx$
*	*	*		

This was repeated until the same twelve problems, in a different setting and arrangement each day, had been given on six different days.

Then the answer of each child to each problem each time was tabulated in order that the various answers of the child to the

same problem might be compared. As the problems were given on six different days each child handed in six answers to a problem. And this set of six answers constituted the unit with which the study dealt. If a child answered a given problem correctly every time, the set of answers was entirely correct. If he answered correctly five days and incorrectly one day, this was called a single error. If he answered correctly four days and said (for example) $3a$, incorrectly, on two days, these were two errors which were *alike;* or they were *repeated* errors. If the child answered correctly four days and said (for example) $3a$ and $2a$ on the other days, these were two errors which were *different*. If more than two errors were made, they were classified as if they had been double errors.

Eighty-nine children were present every day and answered all of the problems, so 1,068 (or 89 times 12) sets of answers were tabulated. Of these 695 sets were entirely correct. And 173 contained only a single error. No data could be secured concerning repetition of errors from either of these classes, so they were disregarded. This does not mean that they were considered without significance, but that they were not used at this time for the study of errors which were repeated.

This left 200 sets of answers in which two or more errors occurred. Of these only 32 contained two different errors. The other 168 contained two like errors, or repeated errors. Or stated as percents, 32 out of 200 is 16 percent of unlike errors, while 84 percent of the errors were repeated.

The above problems were given on six different days. In order to see whether the number of days was a large factor in the results, the above figures were compared with the results which would have been secured at the end of five days only. If only five days are used, there are 1,068 sets of answers, of which 697 are entirely correct, and 197 contain only one error. This leaves 174 containing more than one error. Of these 31, or 17.8 percent, contain different errors, while 143, or 82.2 percent, contain like or repeated errors. That is, the results, practically, are 18 percent instead of 16 percent and 82 percent instead of 84 percent at the end of five days instead of six days.

The above results were secured late in 1923 for only 89 children. The study was repeated late in 1924 and complete answers secured from 211 children. The process followed the

second time was the same as the first time except that less problems were given aside from those problems whose results were tabulated, and except that the time interval between papers was less. This time the results were as follows: There were 2,532 sets of answers, of which 1,621 were entirely correct, and 402 contained one error. This left 509 containing two errors. Of these 108 or 21 percent were unlike errors, while 401 or 79 percent were repeated errors.

If the figures for the first and second group of children are combined, a total of 3,600 sets of answers was handed in, of which 2,316 were correct, and 575 contained only one error. This left 709 containing more than one error. Of these 140, or 20 percent, contained two different errors, while 569, or 80 percent, contained repeated errors.

The above figures are shown in tabular form:

	6 days	5 days	6 days	6 days
Number of children	89	89	211	300
Sets of answers	1068	1068	2532	3600
Correct	695	697	1621	2316
Containing errors	373	371	911	1284
Only one error	173	197	402	575
More than one error	200	174	509	709
Different errors	32	31	108	140
Like errors	168	143	401	569
Percent unlike errors	16	18	21	20
Percent like errors	84	82	79	80

From these sets of figures from a total of 300 children, it would appear that a large percent of wrong answers, where some wrong answer was given more than once, are the beginning or the result of a habit in process of formation.

This statement is in line with statements of Thorndike in the *Psychology of Algebra* (pp. 440 ff.). Here he considers each part of any process as a habit to be formed, and discusses the specific practice needed for its formation.

Rugg and Clark, in *Reconstruction of Ninth Grade Algebra* (p. 88), make a similar distinction when they classify errors

as accidental or recurring. "Recurring errors supply a means of determining exactly which types of problems, operations, or processes pupils have not mastered. . . . Thus in diagnosing difficulties in learning, it is the recurring errors that are significant."

And G. C. Myers made a similar statement concerning mistakes made in arithmetic, in an article entitled "Persistence of Errors in Arithmetic" in the *Journal of Educational Research* for June, 1924. He found the errors made by the children in simple number combinations were not isolated accidents but were repeated from day to day. He concludes: "Let us take errors out of the class of motives and morals and put them in the class of habits where they belong."

It must be borne in mind that the material in this study is all fundamental processes, or drill material. But in the case of such drill material it would seem that errors tend to be habit-forming, bringing a line of repetitions in their train. If this is so, more or less errors will be made according as the teacher places less or more emphasis upon the prevention of the initial error, because that first error is apt to be the forerunner of other similar errors. And, consequently, the attempt to prevent errors, rather than to corect them, is the more economical mode of attack in teaching, even though it necessitates more time at first.

A PROJECT IN MATHEMATICS

Two Algebra Classes Build a Railroad

By DONALD P. SMITH
North Shore Country Day School, Winnetka, Ill.

In this project, two classes in mathematics, one of boys and the other of girls, built and operated a railway system. Since it was impossible to obtain the necessary data on any small line in the vicinity of the school, a branch of the New Jersey Central was used. Enough information was available concerning the actual system here to make the project real and possible.

The work was so planned that while the boys were going through the details of the surveying, excavating, buying land and the actual building, the girls were making a complete study of each of the towns along the line, and the industries of importance, from which they figured the probable yearly income. Each group was organized as a board of directors with a secretary to record the information gathered, and reports as accepted; and with the instructor as president. A general view of the project as a whole was then worked out, and committees chosen to take care of each question, as it arose as a detail necessary for the completion of the project.

In the section computing the probable yearly income of the system, all of the committees combined their efforts at first in obtaining a general knowledge of the size and the type of each of the towns with which they were to deal, and detailed information concerning the amounts of output of the larger industries, and the raw materials necessary for their production. This was accomplished by writing to each of the towns and industries before the Thanksgiving vacation, so that no time was spent in awaiting answers. With this data in hand they were able to get a clear idea of the towns as a whole, and as units of their system. The committees on passenger and freight income were then ready for action.

The incoming freight was divided into the classes of food, coal, manufactured products and commodities, lumber, ice, cement, gravel, sand and raw materials for munitions. The building materials consisting of sand, cement, lumber and gravel were considered on a basis of the growth of each town accord-

ing to the apparent possibilities. The amount of coal was determined by obtaining the actual figures from the industries and two of the towns, and then figuring the quantities for the other towns on a comparative basis. The report on the amount of food was based primarily on the government data, on the amount of food that a person eats during a year, obvious adjustments being made in the case of farming communities. Figures on the raw materials for munitions were based on actual conditions at the plant, and the manufactured articles and commodities were considered in relation to figures compiled by all the railroads in the country, as to the number of tons per person being carried each year, in the country as a whole.

The out-going freight varied for the different towns, but was much more easily figured, because of the abundance of data received from the industries. Three sand pits gave a steady output of white sand, an ore mine was in operation, and a large steel works was turning out quantities of steel products. The farming districts shipped grain, and the lumber districts, their produce. The existing freight rates were found on all of these things, and from them, the freight income for each town was figured.

In ascertaining the number of passengers to be expected, it was necessary to make out a tentative train schedule, to meet the trains on the main line to New York City, since our trains must connect with them. A thorough study of each town was made, in relation to the other towns and to New York, so that the number of passengers going to the city daily could be determined. The number going from town to town daily was reached after a thorough consideration of the demands of the industries of each.

In the group doing the construction part of the project, a small section of the contour of the land through which the road was to be run, was first studied in detail, thus conveying some of the problems encountered there. The profile of the entire system was then placed at their disposal, and the computation of the cuts and fills for the excavation was under way. Three excavation contractors were appointed to submit bids at a set date. The folly of figuring the area and then the volume of each section of the cut and fill was immediately seen, and it was necessary to develop some formula to simplify it. The shape

was known to be trapezoidal, and an engineering handbook helped them with the angle at the base, and the other necessary dimensions. Most of the group did not know the formula for the area of a trapezoid, so they went back to the rectangle, parallelogram and triangle, and built up the whole proof. The idea of finding an average depth for each cut and fill came next, and then they had the trapezoidal formula, $\frac{1}{2}hl(B+b)$. Each of the contractors then took the figures, thus compiled by the whole class, and made his bid accordingly. All of the bids were rejected the first time, and suggestions made by the board, as to the details they wished made more clear. The result shown in the final bid is given below, as an example of the many contracts received during the progress of the project as a whole.

EXCAVATION CONTRACT

Amount of earth necessary for all fills		75,000 cu. yd.
Cuts		39,000 cu. yd.
Amount of extra earth needed for fills		36,000 cu. yd.
Cuts		
39,000 cu. yd. at 500 cu. yd. per day, for 78 days.		
Average cost of steam shovel per month in operation	1,000.00	
Cost per day $\frac{1{,}000.00}{30}$ for 78 days		2,600.00
Extra Dirt for Fills		
5,600 cu. yd. at 500 cu. yd. per day, 72 days.		
Cost per day $\frac{1{,}000.00}{30}$ for 72 days		2,400.00
Hauling		
Average haul of five miles, at rate of $2.00 per train mile. 6 cars per train with 60 cu. yd. per car. 6 times 60 equals 360 cu. yd. per train. 75,000 divided by 360, equals 208 trains. 208 times 5 equals 1,040 train miles. 1,040 times $2.00		2,080.00
Rock Blasting		
3,500 cu. yd. at $.60 per cu. yd.		2,100.00
	$9,180.00	
38 percent adjustment for change in costs, based on increase of actual materials and labor. $1.38 times $9,180.00		12,664.00
Labor		
10 men and one foreman with each shovel. Foreman at rate of $1.00 per hour and laborers at rate of $.50 per hour. Figured on basis of 8 hour day for entire time.		
1 times $1.00 times 8 times 78		624.00
1 times $1.00 times 8 times 12		576.00
10 times $.50 times 8 times 78		3,020.00

10 times $.50 times 8 times 72	2,880.00
10 men and one foreman for leveling.	
1 times $1.00 times 8 times 78	624.00
10 times $ .50 times 8 times 78	3,020.00
Teams for leveling at rate of $6.00 per day.	
4 times $6.00 times 78	1,862.00
Temporary bridges (wooden trestling)	
44 feet at $15.00 per foot	660.00
	$25,930.00
25 per cent factor of safety.	
$1.25 times $25,930 .... $31,412.50	

Along with the excavation contractors were bridge, track-laying and road bed contractors, and all of these bids were submitted and finally accepted. A committee investigated the prices of land, both in the towns and in the country, and produced a report on the cost of the property. There was much discussion as to the advisability of using one or another type of ties, and as to the size of the rails, etc. These questions were decided by reference to a hand book, and consideration of the location of the system, in connection with freight rates on construction materials. The operating cost was not done in so great detail, being compiled directly from the reports of other railroads in an engineering handbook, and properly adjusted.

The following figures were the result, and although they may just happen to have come out as closely as they did, it shows that none of the computations were wild, and unreasonable. There could have been no juggling of the figures because the income was in the hands of one group, and the cost and operating expenses in the hands of another. The annual income proved to be enough greater than the operating expenses to allow an interest of 8.6 per cent on the investment.

Total Yearly Income	$555,342.23
Total Operating Expense	449,216.98
	$110,125.25

The project occupied just one month of the time of the class in school, being so arranged that it started just before the Thanksgiving vacation, and ended just after the Christmas vacation, thus allowing these times for letter writing in search of information. The very evident result was in the new interest in

mathematics as a whole. The real drive aroused by the immediate interest did not die out as soon as the project was finished, but continued for the remainder of the year, in the case of most of the pupils. It served to get some pupils going in their mathematics, who had simply been hanging on before. The direct learner found that all of the stuff he had been exposed to might have a real place, and he kept looking for that place with all of the work for the rest of the year. To help him find it was rather difficult in some parts of our high school algebra.

The letters written to the towns and to the industries were not only a real help to them in getting their ideas and questions in good form, but the answers they received were a revelation to them. The industries sent them very neat and complete reports on the data they wished, and did it with a courtesy that showed interest. The information received from the railroads and the government bureaus was of the same calibre. These combined to give them the idea that people were interested in the educative process who have no direct contact with it, and that courtesy is not theoretical, but real.

It gave them a fundamental idea of the methods of operating and financing a big business project, and a glimpse of the organization necesary. They gathered a real concept of the meaning of a dollar and the meaning of a million dollars, and the application of these terms to an actual business operation. In summing up all of the figures for the construction cost and the operating expense, they saw the proportions in a way that will stick, and although they realized that the 8.6 per cent may have been largely by chance, they could use that as a figure and compare our dividends with those they would have reviewed had the money been placed in bonds or stocks; and the comparative risks.

The pupils spent one month in what they could realize to be a practical situation, and hooked it up with their mathematics in a way that gave them a much broader outlook on its place in the world in which they were living; an outlook which they could never have gotten otherwise.

ADAPTING PLANE GEOMETRY TO PUPILS OF LIMITED ABILITY[1]

By MARTHA HILDEBRANDT
Proviso Township High School, Maywood, Ill.

I wonder how many teachers really look forward to teaching a class of pupils, made up of those who are usually conceded to be slow to grasp a subject—especially if that subject is geometry? Not very many, if the comments of the average teacher of mathematics may be used as a basis for forming this opinion. I will frankly admit that I accepted my first such class in plane geometry with many misgivings. However, as the years have rolled by, we have had many such classes and I have had to teach my share of them.

What kind of pupils does one usually have in such a course? I have had more than one class in which about one-half the pupils had taken the subject before and failed and the other half had repeated one or both semesters of algebra, two or even three times before managing to obtain one credit in the same. Other classes were made up of pupils whose intelligence scores were low and who in the opinion of all teachers in the mathematics department, judging from previous work done by these pupils in the department, were the kind who would have much difficulty in grasping geometry. But this is only a superficial analysis of the type of pupil present in the class. Closer study enables one to classify them according to the difficulties they encounter in mastering their work.

Very few are really so low in ability that they are unable to grasp anything. There is the pupil who seems very slow to understand and may appear to work long and hard, but much of his work is a waste of effort. He has never had the experience of really understanding a course day after day. Others, although they have less than average ability, suffer from mental laziness. This type may be active physically and sometimes clever where nonsense is of chief importance, but he has a tendency to shirk wherever work of the mind is required. He refuses so consistently to use the mind, that he appears very stupid.

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[1]Read before the Mathematics Section of the University of Illinois Conference at Urbana, Nov. 21, 1924.

Sometimes this mental shirking is due to the fact that the mind has been dulled by an excess of mechanical study. Haven't you had pupils come to you and request some mechanical means to be used in preparing a lesson rather than to be compelled to rely upon their own minds to think them through the assigned task? Such pupils are quite capable of solving a problem and making some kind of a recitation, going through all the motions, without realizing the whys and wherefores or even the plain common sense of the process. Some of these pupils readily memorize things without having the slightest understanding of the subject matter memorized. Another pupil of this mentally lazy type is the one who prepares his work spasmodically, the pupil of irregular habits, who is a failure because he habitually persists in leaving so much work partly finished, that the whole course is incomplete and appears to lack meaning. Then there is the pupil who is not accurate in his use of the English language. He is unable to read with understanding and does not say exactly what he means. This may be due to carelessness, sheer mental laziness or lack of mental effort. Or the child may be handicapped because of the foreign language spoken at home —sometimes very poorly spoken at that. Then you have the child whose home conditions are such as not to be conducive to good work. At the present time a high school education is practically within reach of every child—and many parents send the child through school expecting the extra schooling to provide the child with cleaner and less difficult work in later life than would otherwise be the case. They have very little understanding of and no sympathy for mental work. They do not realize that a child may sit with a book for two hours and in reality hardly accomplish ten minutes of work. Not infrequently such a child is provided with no definite place for study. I have had children who worked in some store from four until ten o'clock every day, all day Saturday and sometimes Sunday and came to school during the day to get what they could and rest, probably. So, in reality there are three types: the child who has some physical or financial handicap; the child who is really born short and needs special presentation of material to meet these needs; and finally the child who has less than average ability but is mentally lazier than he is stupid. I shall in this

discussion turn my attention only to the latter two types as the former would naturally fall into one of these, if his other handicap were removed.

A first thought it would seem that geometry would be of little value to such people. Surely they will never need the actual subject matter of geometry to earn a living Quite true, but most teachers of mathematics will admit that the end of mathematics teaching is power and not knowledge. What does geometry require of the pupil studying it? We are told that it requires the ability to read with understanding, to visualize that which has been read, initiative to use acquired facts, arrange them in logical order and thus reach a definite conclusion. It requires reflective thinking and understanding rather than memory. It requires the ability to express these thoughts in a definite, clear cut and precise manner. This does not mean ease of expression or fluency in speaking, but the ability to select the words which will exactly express thoughts rather than those which vaguely indicate a multiplicity of meanings, some of which are undesirable and untrue. It requires the ability to try again when one method has proved a failure until eventually success is obtained. Aren't these the very abilities which a child of this particular group needs developed to the fullest capacity he is capable of attaining?

Let us now turn our attention to the attitude of this class toward geometry. Many pupils are afraid of the course, in some cases hopelessly so. They seem to feel, in fact they know, because their parents, brothers, sisters, relatives, neighbors or friends have told them, that geometry is a difficult course, of little practical value and one in which so many fail that they might as well make up their minds never to understand it. In questioning them, I find that one of the chief reasons for registering for it is, because two years of mathematics are required if mathematics is presented as a minor and having exposed themselves to algebra, when they didn't know better, it is now necessary that they swallow the second half of the bitter pill. I find it a good plan to dispel this attitude as rapidly as possible. Until they have attained some confidence in their own abilities, I give them work, which they are capable of doing with great enjoyment and which is at the same time essentially geomtrical in character.

The textbook we used, when we first began segregating our classes on the basis of mental ability, was of little value to me for work of this kind. It began with several pages of definitions, a few pages of exercises, many of which required algebraic solutions, a page of explanation of geometrical symbols, and then plunged directly into demonstrative geometry with the theorem stated, figure drawn, proof written out in full, possibly omitting a reason here and there and putting the word "why" with a question mark after it instead. So I collected and made up material which could be adapted to the needs of my classes and filed it away for future use. As I find new material, I add it to my file.

Nowhere in our system had any intuitive geometry been taught and I decided to use it to develop interest, curiosity, some thought and familiarity with geometrical terms and tools. The first few days of settling down are used to develop a background for the course, especially by delving into the history of its development. Now our high school library contains several histories of mathematics, but most of them are not exactly written for the high school pupil—especially the pupil of limited ability. However by giving definite references to particular pages and paragraphs and filling out this reading with a well rounded class discussion, and with the aid of some framed pictures of mathematicians hanging in each of the mathematics rooms of the school, it is possible to create an atmosphere of interest. While the class is still under this influence we begin to use our tools in simple constructions. This can be made to mean much to the child. First of all he is given definite directions. These he must read with understanding and carry out in detail, step for step. I frequently send a child to the board to make such a construction and to talk while he is working, explaining what he is doing and why. This familiarizes him with the accurate use of the new geometrical terms he is acquiring in his vocabulary. It also impresses the construction more firmly on his mind, for it is difficult for the pupil to explain a process which he does not understand clearly and at the same time it may help to drive away difficulties of other members of the class. Sometimes one pupil is sent to the board and another directs his movements. If the pupil at the board finds it possible to misinterpret the directions and yet literally obey them

he is privileged to do so. I find my classes willing and eager to correct mistakes made by the pupils reciting, and always take advantage of this opportunity to impress upon their minds why one method is incorrect and another correct. This discussion frequently leads to the discovery of simple postulates and axioms which we promptly label and file away as general truths which may prove valuable at some future time. During this period, geometrical terms are thoroughly explained and used, but definitions are never learned. Lists of graded exercises are also a great help in familiarizing a child with the exact use of these technical terms and should not be used sparingly. This preliminary work must be thoroughly planned but I find that there is a tendency to make it too simple and thus lose the interest of the pupil. Care must be exercised that the work is planned so that the pupil realizes that he must be constantly alert in class and at work.

In this manner it is possible for the student to assimilate many fundamental geometrical notions, become familiar with technical terms and learn the correct use of compasses, straightedge and protractor. I even proceed to teach the work on the congruency of triangles having two sides and the included angle or two angles and the included side equal, using the superposition proof as if it were neither new nor different. We construct the triangles as required, cut them out, place them on each other, see that they fit if the corresponding parts are placed on one another, and proceed to analyze how it was done and justify each step.

But after these theorems, the work is demonstrational geometry begins in earnest. This next period is a more or less difficult one, in spite of all the careful preparations which have been made during the first few weeks. The newness of the subject and of the use of geometrical terms is beginning to wear off. The boy or girl with the lazy mind is beginning to resent the idea that he is expected to think throughout the year. But if I can make him think for himself and prevent him from temporarily memorizing the text regardless, then I have fought a good fight.

The class must be convinced that proofs are necessary and facts cannot be accepted because they appear to be true. It is an easy matter to get plenty of material on illusions and with

little encouragement, pupils are able to make up examples of their own and use them on each other. At this time it is also necessary to introduce the value of a neat, well-appearing proof in good general form, and the fact that every theorem or problem must be read with understanding for three things, (1) a figure, (2) the hypothesis, or definite material given to work with, and (3) the conclusion or definite thing which it is our task to prove true. With a little special attention on this work, pupils are soon capable of drawing their own figures, regardless of the figures in the text, and stating definitely and accurately what is given and what they must prove.

Now for a class of limited ability, I find that many texts offer too many theorems, some of which are after all irrelevant and unimportant. Therefore I choose a minimum requirement of the most important theorems. These the class must master,—i. e. be able to prove and use readily in the proving of any number of exercises built around them.

I pay very little attention to the textbook or its presentation of the proof in any of the beginning theorems and strongly advise the child not to depend upon the text in his study. It seems to me that this is one of the best ways of preventing useless memorizing and of developing reflective thinking. Suppose a theorem is to be presented to the class. Perhaps the fundamental idea of the theorem has occurred to the class in connection with some other piece of work. The statement is placed on the board, read and analyzed. Some one volunteers to draw a figure. Classes can be made very critical of these figures. For example if the statement speaks of any triangle and the figure drawn seems too nearly isosceles or equilateral, the class will not accept it. Likewise the class demands accurate statements of what is given and what must be proved. Then we proceed with the proof. At first I teach them to analyze the situation by asking leading questions, but I try as soon as possible to get them to the place where they can ask themselves these questions and so arrive at the proof alone. Here is another type of recitation I sometimes use. I construct some figure on the board, fitted to a theorem or problem which has not been stated to the class. The class observes this work carefully and then some one volunteers to state what has been given. Some one suggests that possibly something else may

be true and every one tries to prove this particular conclusion false or true. If a proof is attained the class then attempts to state a theorem in as good English as possible and usually is very much pleased at its success.

In order to keep full attention it is a very good plan to make each pupil critical of the one making the recitation. It requires some study and some grasp of the work to be able to criticize a recitation, moreover a class makes a pupil giving an incorrect or careless criticism most uncomfortable and finally no pupil relishes adverse criticism from his peers and so will do his best to make corrections impossible.

The more familiar a pupil is with the theorems, he has had, the easier it will be for him to call them to mind whenever he needs them. To attain this degree of familiarity I use two devices. We develop a kind of mental filing system. When a theorem is discussed we label it definitely according to its purpose, i. e. it can be used to prove angles equal, triangles congruent, or line segments equal. Then it belongs to this particular class of theorems and occasionally we sum up all the theorems which will help us prove this or that fact. This summing up is easily done in analyzing a proof. Then I also frequently give the class lists of graded exercises which demand the use of the theorems in their proof.

These graded exercises are really very valuable in more ways than one. It is possible to introduce many practical applications of geometry through them. They strengthen the child's ability to think because he has no prop like a completely written out text-made proof to lean on, as is the case in theorems, and finally they stimulate the class to greater activity. Even though the group is below average in mental ability, some of its members will always excel others. I keep this in mind in the assignment of exercises. They are graded in difficulty and a certain number are required for a passing grade in the course. If, however, the pupil has not used up the allotted forty-five minutes of study when the assigned exercises have been finished, I strongly recommend that he do as many of the remaining exercises as possible. This helps him to attain a grade above the average. You would be surprised at the number who avail themselves of this opportunity and of the pride they show in being able to do more than the required amount. Sometimes,

more than one proof is possible for an exercise. In this case students are sometimes skeptical of other proofs as well as their own. This is splendid for class spirit and interest.

One of the chief difficulties with classes of limited ability is that their study habits are poor or incorrect. For this reason I sometimes present an assignment and when I think they have it completely in mind, I give them about fifteen or twenty minutes of class time to begin their study. Then I watch for the time-wasters and those who are careless about getting the assignment correctly and give them my personal attention. I also encourage those who come poorly prepared, to stay the seventh period, a special help period in our school, and do their work in the mathematics room. This gives me more of an opportunity for individual corrective help and often gives them a better atmosphere in which to study.

I discovered last week that some of my pupils were of the impression that I received "time and one half" for giving them this special attention after school and that was my reason for inviting them to come in. This "time and one half" idea and all that goes with it is one which makes some of these pupils of limited ability appear so stupid. Their attitude is approximately this: a job is assigned, it must be done, well or poorly makes little difference. Union hours must be strictly observed, loafing on the job is permissible, in fact it would be treason to accomplish too much and extra work is not to be thought of. To give to these people a professional attitude, impress them that only the best work is worthy of them, that time does not count, only the joy and the pride of a job well done is worth while, this task requires much tact, time and energy.

Accuracy in all things especially in speech is another thing we strive for. If a theorem is stated incorrectly, the pupil must show that it is incorrect and why. Merely admitting it, is not sufficient. If the pupil's statements are ambiguous, he can be grossly misinterpreted and so be taught to be more accurate. Classes are quick to do this and it keeps their minds on the alert.

I could speak of many other things, how we treat the construction of loci to clinch the interpretation of theorems, how we use algebraic problems to help make the work more practical, and assign plates for construction work to develop motor

skill. But after all the problem of the pupil of limited ability really resolves itself into this—just how human is the teacher, how willing is she to study, help and give of her energy to each individual pupil, has she the personality and ability to get the pupil to develop and use such a mind as he has to its fullest capacity in spite of himself? One other question arises. These people of limited ability, do they ever amount to anything? Is it worth while? Doesn't teaching them to make the greatest use possible of their mind, to realize the benefits to be derived from the power to think, make them better citizens? Isn't it one method of conserving waste? Isn't it often a step to something finer and better for them in this world?

THE REACTION OF A REACTIONARY

A rhythmic writhe of rage

Dear Editor:

Your authors and contributors have long inflamed my wrath by the silly-willy adjectives they now apply to math. It's "animated, motivated, antiquated, decimated, simplified, or unified, exemplified, or modified, humanized, or vitalized, harmonized organized"

Don't deem me just an unkind scoffer. For authors new these types of math I humbly offer: "cerebrated, acclimated, elongated, elevated, cultivated, ruminated, barricated, cauterized, uncriticized, vulcanized, unsubsidized, galvanized, or unrevised, evanescent, or florescent, or quiescent, effervescent, petrified, or putrified, mummified, undignified, magnified, or atrophied, mystified, or bonafide." The list is short. 'Tis hard to pick the worst, yet these should plenty be, 'til April 1st.

Ed. To my Litany in church I frequently add thus, "From all those awful adjectives, Good Night! deliver us!"

Moral

Forget slick, trick nicknames we should.
The stuff by any other title's just as good.

Yours very sincerely,

FREDERICK A. KAHLER.
New Trier High School,
Kenilworth, Ill.

ARITHMETIC IN THE JUNIOR HIGH SCHOOL[1]

By LEWIS W. COLWELL
Principal, Grover Cleveland School, Chicago

The curriculum of the junior high school must be determined on the one hand by the needs of a developing civilization and on the other by the nature and capacities of developing youth. These two criteria of worth are by no means opposed to each other. They constitute no bifurcated demand. They set up no dilemmas. For every child is born into organized society on the one hand and becomes a duly constituted member thereof, while on the other hand he possesses a social nature that fits him into the world's work just in the measure that he finds himself. It is perhaps not far afield to say that all friction due to anti-socialistic tendencies is a maladjustment of individuals who have not discovered what they are good for.

To select a school curriculum we must study the problems that civilization is solving and the work by which society maintains itself, with a view of discovering the appeals that training can make to the capacities and tastes of growing and expanding life. When we examine industry, we find it full of quantitative problems without the solution of which at every step of the way no satisfactory progress is possible. Economies of production, of exchange, of construction call for constant investigation and calculation. To reveal some of these data to the inquiring mind of youth is to enlist their instant interest and to prepare them for perseverance and endurance though the solutions demanded entail protracted effort and sustained attention. Young people are naturally interested in whatever makes the wheels go round and will spend themselves eagerly in the effort to know and to control dynamic things.

The traditional content of the textbooks of seventh, eighth and ninth grades is notoriously unfit for adolescents in some important respects, among which are the inclusion of material foreign to their interests and the introduction of extended and complicated calculations in order to make problems different from and more difficult than those of the lower grades. Much

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[1]An address delivered at the Central Association of Science and Mathematics Teachers, Chicago, 1924.

of the work for the seventh and eighth grades has been cast in the same mould as that of the lower grades, differing from it only in degree and reiteration of dreary, useless complication.

Pupils leaving the sixth grade have been taught the elementary processes with integers; they have some knowledge of common fractions; they have been taught a little regarding decimals. To continue this kind of work and to introduce extended and complicated business practice under the pretense of teaching practical applications of percentage corresponds to no felt need. If pupils do not become apathetic or rebellious under such enslavement, it is because they have initiative smothered and so are content to plod unwillingly and unwittingly but obediently along the profitless way, regaling themselves perhaps by drinking of forbidden streams, or at any rate never discovering the delights that may be theirs.

Nor is this sin against childhood any less in the usual ninth grade algebra. Pupils are plunged into abstractions for which they have not been prepared by experience and insight, so the floundering increases and the mortality rate is so high as to deserve the name "the slaughter of the innocents."

Now, seventh grade pupils deserve a better fate than to fall into the type of mathematics usually imposed upon them. Nature and industry are full of interesting problems making a quantitative appeal. Boys and girls move through a world of the mechanical; they see material construction on every hand; they behold forces everywhere operating whose control is an application of the laws of quantity. Somebody or something is constantly arriving or departing. Reduced to their simplest terms, the problems that arise in this field are questions of space and time and motion and it is high time when pupils enter seventh grade that they be led to investigate the spatial elements of this busy, breathing, interesting world in a new and definite way. It is time to acquaint pupils with some of the elements of precision in measuring, to train their powers to estimate distances and spaces. In a word, to develop within them *quantitative appreciation.* Work with numbers merely cannot develop the mathematical sense; only the hand-training and eye-training that come from manipulation of spatial forms, solids, surfaces, lines and points can furnish the basal experiences upon which the significance of mathematical value can rest. Numerical evaluation

gets its meaning when and if used as subordinate to these more primary percepts.

Then let us give seventh grade pupils *mathematical instruments.* Let them learn the use of straight-edge and compasses and protractors and linear scales including the metric linear scale—not by studying about them but by using them as tools of investigation.

This would mean of course giving seventh grade pupils exercises in actual measuring with linear scales, using not only the common units yard, foot and inch but the rod also as well as metric units. A clear concept of the mile may be reached by exercises appealing to the constructive imagination and by drawings made to scale. Thus the conception of distance should be enlarged and elaborated on.

Pupils should be taught how to draw to scale after discovering the need for drawing to scale in platting and mapping first familiar near-at-hand areas larger than the paper on which the drawings are made and, later, by representing larger areas and remoter distances. This will introduce some real and significant calculation with numbers having both intrinsic interest and immediate application. The interpretation of blue prints is suggested. The application to geography and varying scales of maps is evident. In geography especially the educated sense of distance is essential.

In mapping and planning there are three characteristic problems that appear. (a) If we are drawing to scale, we have given distances and a given scale. Any given exercise then is a series of constructions dependent upon calculating the required line segments from the given data, viz., the actual line given to be represented and the given scale. Representing these elements by L, s and l, we have given each time L and s to find l. Each operation is observed to be multiplying a given distance value by a given ratio or scale, hence we may generalize the operation by writing the formula: $l = Ls$. (b) If we are using a map and a given scale of miles to find the actual distance between two cities, we have given l and s to find L. In this case we must divide the measured distance on the map, l, by the ratio s (or multiply it by the reciprocal of s). Generalizing we write $L = l \div s$. This is division by partition. (A term value divided by a ratio.) (c) A third problem may arise. Having a given

plat or map and the actual thing or distance represented, we may desire to find the scale. In such case we must divide the platted distance l by the actual distance L. This time the formula becomes $s = l \div L$.

These three formulas with appropriate changes in letters will be recognized as typical of the three so-called cases of percentage. By substituting B, P, and R, for L, l, and s the formulas become (a) $P=BR$, (b) $B=P\div R$, (c) $R=P\div B$.

Again, if in any multiplication problem we call the product P, the multiplicand M, and the multiplier m. We have (a) $P = Mm$, (b) $M=P\div m$, (c) $m=P\div M$.

If in any division problem we call the dividend D, the divisor d and the quotient q, the formulas may reappear as (a) $D = dq$, (b) $d=D\div q$, (c) $q=D\div d$.

A study in angles and triangles requires the compasses and protractor. The properties of right, acute, and obtuse angles, of 45° angles, 60° angles, are soon discovered. The method of dividing a circumference into sixths, into twelfths, into fourths, and the consequences that flow from these constructions, the bisection of a line, and the construction of a line perpendicular to, or parallel to another are experimented with. In such a mathematical laboratory, experiment is followed by application and generalization and thus a real sense of mathematical values begins to find lodgment in pupils' heads.

Numerical calculation falls into its legitimate place as an evaluation of quantitative relationship, so that that laborious ciphering which has hitherto been a forbidding and unwelcome taskmaster takes the role of ministering to a felt need. Hence pupils gladly endure necessary drills because of the help they secure from the additional skill they acquire.

Pupils of the seventh grade are usually far from skillful with figures and facile in numerical operations. They need little additional instruction in technique, perhaps, if the work of the preceding grade has been well done, but they need loads of practice in direct and simple calculation, as in ordinary adding of columns. They need gentle leading in the snares of ordinary subtraction, in multiplying, and in long division. Measured graded daily practice, progressive in character, is needed in most if not all classes. This should be abstract in character and emphasis should be laid on accuracy, though facility and prompt-

ness should also be encouraged. Greater familiarity with common fractions and with the handling of decimals is also desirable and should be provided.

In the advanced seventh grade, there is a fine opportunity to apply drawing to scale to the making of simple graphs, line graphs, and bar graphs principally, and to open up the mysteries of percentage through graphing and diagraming. The study of percentage should return to whatever knowledge of decimals may have been previously acquired and should seek to illustrate and extend the same. Multiplication by pure decimal multipliers needs careful attention. The product of such a multiplication should be sensed as of less value than the multiplicand. This is essentially the first case of percentage and may be the basis for instruction in the same.

Division of one number by another resulting in a pure decimal leads up to the so-called second case of percentage. The observations should be developed that division of whole numbers is not limited to cases where the divisor is less than the dividend, that the quotient need not be greater than unity, and that any quotient is really the expressed ratio of dividend to divisor. This is the second typical problem, "Given two terms, what is their percent relation?" Another form of this problem is "What part of a given number is another given number?" Such examples throw light on this case of percentage. This should be seen as a measuring operation involving (1) a standard or measuring number, (2) a measured number, and (3) a resultant of measuring, or ratio which ratio or quotient in such cases is most frequently less than one.

Division of a whole number by a proper fraction or a pure decimal typifies the third so-called case of percentage. Previous to the experiences that arise out of such percentage problems, the solution of such division problems has probably had only a superficial, formal meaning. "Finding the number of which a gvien number is a given part" is the significance of such division. "Finding a required number when a given part of it is given" may be done by dividing the given part by the given fraction (or multiplying by the reciprocal of the given fraction). This so-called indirect problem is not easy to grasp and is not stressed by some teachers, but it seems important to the writer because it is so significant. It is really division by partition,

so-called. The previous type or Case II is division by measure, so-called.

In the advanced seventh grade daily practice with fundamental operations, fractions and decimals, should be continued as in the beginning seventh grade.

For the eighth grade, I would suggest a return to the experimental, constructive exploration of spatial form. First let such pupils study the kinds and properties of triangles and practice the various constructions and mensurations involved, then let them proceed to study quadrilaterals in the same fashion and perhaps to make some of the regular polygons and calculate the interior angles of the same. This will lead naturally and easily to the study of parallels, transversals, and perpendiculars. The mensuration of the circle and of the cylinder might follow, but it seems to me that the mensuration of the pyramid, cone, and sphere should be postponed for more advanced work.

By the time the advanced eighth grade is reached, some simple algebra might be systematized by gathering up formulas developed and used in the mensuration work already accomplished. If the convenience of letters as symbols of generalized or abstract numerical values is emphasized, then from this experience it should be an easy step to sense the convenience of representing any undetermined number by a letter and to translate the conditions of a simple algebraic problem from ordinary verbal language to the equational or mathematical form. The solution of such equations by a series of perfectly evident easily taken steps follows and thus, without undue effort, the algebraic *pons asinorum* has been crossed. These derived equations, I hold, should be reached directly and intuitively rather than by the time-honored but stale device of axioms and laborious expositions of feats of balancing, of adding or subtracting equals. This is usually so formal and far-fetched, at this stage of the pupils' mathematical evolution, as to obscure direct relations that are easily seen. It is likely to substitute a sort of juggling for direct insight. Such contrivances may or may not be of some value when more advanced equations are reached but the purpose here is not so much to develop manipulative skill as to produce real insight into mathematical situations.

After pupils learn to translate from common language to equation form many simple algebraic problems and to reach an

algebraic solution of such problems, some practice for them is legitimate that will fix the knowledge which has been acquired and stress simple manipulative skill. Such forms as the following are recommended.

$3n = 12$	$6x - 2 = 4x + 14$	$x/5 = 8$
$36 = 4b$	$140 = 8x + 4$	$2/3x = 16$
$3n + 5 = 14$	$20m = 40 + 12m$	$3x/4 - 2x/3 = 2$
$7m - 8 = 43$	$8n + 4 = 6n + 10$	
$6x = 15 - 3x$	$13 + x = 450$	

Verification of solutions by substituting in the first equation of a solution the value found in the last, will lead naturally and easily to evaluation. The use of formulas in mensuration is really evaluation. So is the use of the percentage formula $P = RB$, the interest formula $I = prt$, the cost formula $c = pn$, and the selling price formulas, $s = c + g$ and $s = c - l$.

This is perhaps as far as an eighth grade class should go.

A study of similarity of triangles and of other polygons and of solids develops the subject of ratio and proportion in a clear and graphic way. From the study of ratio and proportion as applied to the geometrical elements, further study of the subject as applied to other quantities, time, capacity, weight, money value, etc., is easily made and because of its analogy to the preceding study is easily understood.

Simple studies in indirect or calculated measurement by means of similar triangles, such as heights of buildings, and width of streams, can be introduced here. Heights of trees or of poles may be calculated by comparing the shadow with a shadow cast by an object whose height is known or it can be done by an observation on the ground at a given distance from the pole, which observation determines the angle of elevation and includes a staff of a given length with its top in the line of observation, or, once more, the height can be determined by platting the given distance on the ground and the angle of observation and then measuring the platted height that results. This can be reduced to actual height by means of the scale used.

All this is within the capacity of eighth grade pupils that have had the previous seventh grade course here laid down. It is suggestive, illuminating, deals with familiar elements of pupils'

everyday experiences and is much more interesting than solving problems in bank discount or the usual mercantile type problem which most textbooks spawn in such tiresome abundance.

A study of the relation of the hypotenuse of the right triangle to its other two sides, introducing not only the triangle whose sides are in the ratio of 4, 5, 6, but others with the ratio 5, 12, 13, or the ratio 8, 15, 17, or the ratio 7, 24, 25. Such a study with accompanying drawings to scale motivates the topic of square root and furnishes an introduction to it that is clear, definite, and full of suggestion.

Since some business practice will be insisted on and since it has practical value, some attention may properly be given it, but it should be remembered that such practice is not properly mathematics, useful though it may be and is. Keeping accounts and the making of budgets are worth some time and attention. Likewise the topic of banking and familiarity with the methods of depositing, withdrawing, and borrowing or investing money and calculation of interest are commercial topics of value and worthy of attention.

Daily practice in calculation with progressive exercises designed to increase accuracy and facility, should be insisted on unceasingly. Short processes of calculating should be learned with some care. The use of short cuts to correct results should be encouraged. The utility of canceling common factors in examples that involve such possibility should be remarked.

No effort is made in the foregoing discussion to indicate work for ninth grade. Such an outline when it appears, must deal in part, perhaps largely, with algebraic conceptions and solutions and matter resting on the same. The arithmetic of the junior high school must therefore be contained in large part somewhat as outlined above in the seventh and eighth years of school.

REPORT OF THE SEVENTY-NINTH MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

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The seventy-ninth meeting of the American Association for the Advancement of Science was held at Washington, from Monday, December 29, 1924, to January 3, 1925. Events of mathematical significance will be described briefly.

The American Mathematical Society held the first meeting on Monday afternoon, December 29th. Two sessions were held on Tuesday; on Wednesday morning the Society held a joint session with the Mathematical Association of America and Section A of the A. A. A. S.

This annual meeting was especially marked by the second award of the Bocher Memorial Prize, for a memoir published in the Society's *Transactions*. The prize was divided equally between Professor E. T. Bell, of the University of Washington for his paper entitled *Arithmetical Paraphrases* and Professor S. Lefschetz, of the University of Kansas, for his paper *On Certain Numerical Invariants of Algebraic Varieties with Applications to Abelian Varieties.* The second, Josiah Willard Gibbs lecture, was delivered by Robert Henderson, Vice President of the Equitable Life Insurance Society, on Tuesday evening; the lecture was *Life Insurance as a Social Service and as a Mathematical Problem.*

During the meeting of this Society 56 papers were presented by 39 authors representing eleven States. The meeting was the largest in the history of the Society. Professor G. H. Birkhoff of Harvard University was elected President for the ensuing year.

The Mathematical Association of America held a joint session with the Mathematical Society and Section A of the A. A. A. S. on Wednesday morning. Two papers were presented, one by Professor Oswald Veblen of Princeton University on the *Foundations of Geometry;* the other by Professor Harris Hancock of the University of Cincinnati on the *Foundations of the Theory of Algebraic Numbers.*

The Wednesday afternoon session was given over to the consideration of fields of research in economics and general analysis. The Thursday morning session was held in conjunction with Section A, B, and D of the A. A. A. S. of the Mathematical Society. Two papers, *Stellar Evolution,* by Professor H. N. Russell of Princeton University, and *Is the Universe Finite?* by Professor Archibald Henderson, University of North Carolina, were read.

Thursday afternoon the members of the Association were invited to attend the History of Science Society Section of the A. A. A. S., at which two papers, *Leibnitz, the Master Builder of Mathematical Notations* by Professor Florian Cajori of the University of California, and *Benjamin Peirce* by Professor R. C. Archibald of Brown University were presented. After the conclusion of these two papers the Association adjourned to its own room for its annual business meeting and the presentation of three additional papers. Professor J. L. Coolidge of Harvard University was elected President for the ensuing year.

A joint dinner of the Society and the Association was held at the Franklin Square Hotel on Thursday, January 1, 1925.

The Phi Mu Epsilon Mathematical Fraternity, a fraternity formed in May, 1914, at the University of Syracuse for the advancement and discussion of mathematical topics. This fraternity held one session on Monday afternoon which was followed by a dinner. Membership of this fraternity is based upon scholarship; the fraternity has now nine chapters located in the University of Syracuse, Ohio State University, University of Pennsylvania, University of Missouri, University of Alabama, Iowa State College, University of Illinois, Bucknell University, and in the University of Montana.

On Wednesday afternoon a joint dinner of Phi Delta Kappa Fraternity and Section Q of A. A. A. S. was held at the Y. M. C. A. Following the dinner Professor Charles H. Judd of the University of Chicago delivered an address on *Methods of Cooperative Research in Education.* During this address he brought out very forcibly the failures of a number of national committees to establish their conclusions by experimental evidence. He also pointed out that these committees were too prone to substitute round table discussions for actual experimentation.

The National Committee on Mathematical Requirements Report was one that he attacked.

The Metric Association held a number of meetings. Some of the papers read were as follows: *Teaching Metric Weights and Measures; The Metric System in Our Public Schools; Model Metric Lesson, with Illustrations; Labeling Groceries in Metric Terms,* and *Introducing the Metric System in the Factory.* On Monday afternoon a visit was made to the United States Bureau of Standards which included an inspection of the national prototypes, the meter and kilogram, in the underground vaults of the Bureau. A dinner was held on Tuesday evening.

During Thursday afternoon session of Section Q of the A. A. A. S., Flora L. Scott of the Cleveland School of Education gave a paper entitled, *The Recurrence of Errors in Algebra.*

NEWS NOTES

Mr. J. O. Pyle of the Harrison Technical High School (Chicago) has recently published, with P. Blakiston's Son and Company, an experimental edition of a textbook in plane geometry. In that text, according to Mr. Pyle, the "solution of actual space problems by live students is given first importance. This is because problem solving is the more difficult and more meaningful for the student's life. Deductive reasoning is given that emphasis it merits and not more. What proofs are written out in full, are written with care, and are given as examples of what rigid proofs should be; sample arguments of all the different types usually occurring in elementary geometry. The author claims to have removed that temptation to students to waste time in committing to memory the proofs of theorems known since the time of Euclid. Such memory exercise is as good as memorizing any other bits of classic literature, but not better. The experimenting and pioneering that leads to the control of experience and to the discovery of new truth is eminently more significant than the ability to recite from memory the deduction by some great thinker of a well known fact from simpler principles.

"The real use and significance of deductive proof as the best instrument the mind has discovered for testing plans ahead of

actually trying them out in experience, is emphasized and thoroughly taught. Students are led to see that there is nothing to prove until someone has suggested an answer to a question or discovered an idea. Demonstration is the last and easiest part of the consideration of any live problem. The ability to construct a demonstration is not nearly so easily attained by memorizing somebody else's proof as by forming the habit of justifying thought step by step as it proceeds."

A committee of the College Mathematics Teachers of Ohio greets incoming high school freshmen with the following statement:

Next fall you are going to enter high school and many of you will have to think over the very serious question of the course you will choose and the subjects you will study. As college mathematics teachers we are interested in your problem and want you to consider carefully whether or not you will study algebra and geometry. We hope that you will not let chance or prejudice influence you and that this letter may help you to a wise decision.

1. "How" and "why" are words which you have often used ever since you first began to think and show a keen interest in life and objects about you. Algebra and geometry are going to make you realize very keenly the need and value of asking questions and seeking correct answers. You will learn to do such things as you did in arithmetic, not just by rule, but to think about them and how and why they are so done. As a result arithmetic is going to mean more to you. In addition you are going to learn many other interesting things about numbers and about geometric figures, such as the circle and the triangle, which every one should know no matter of what occupation or calling.

2. In the study of mathematics you will learn to understand and appreciate the value, meaning and use of graphs, symbols and formulas. You have often met them in reading magazines and books. To business men and women a knowledge of them is indispensable, to others very useful and interesting.

3. If you are interested in things which people call practical, such as measuring land, constructing buildings, sailing ships, generating electrical power and applying it to its many uses, manufacturing goods, or building roads and bridges, you may be sure that he who would advance far in such work should have a knowledge of mathematics.

4. At your age you are probably in doubt what you are going to do after you leave high school. Some time you may go to college and then you will find that you will be handicapped if you have not studied mathematics in high school.

5. Do not be afraid to study a subject because some one tells you that it offers some difficulty. The lives of great men tell the story of persistence.

You will generally know, too, in mathematics whether you are right or wrong, which brings much satisfaction.

We hope you will study algebra and geometry and find in them pleasure and inspiration and that some day we may find you in our classes.

Very sincerely yours,

THE COLLEGE MATHEMATICS TEACHERS OF OHIO.

The Ohio section of the Mathematical Association of America sent the following communication to high school seniors in Ohio, in May, 1924:

You may be going to college next year and, if so, you should now consider the advantages of electing courses in mathematics. The mathematics teachers of Ohio Colleges hope that you have been successful in your algebra and geometry, and recommend that you seriously consider the continuation of mathematics in college for the following reasons:

1. It is desirable to know enough about mathematics to participate in conversation, to understand references, and, in general, to have perspective on the matter when terms or ideas from mathematics come up. This field of study, in which law reigns more perfectly than in any other, has thrilled and still thrills some of the greatest men of all times.

2. Certain courses in mathematics are frequently set as prerequisite to getting into some of the other college subjects, and in many of the professional and technical schools. Trignometry is indispensable for engineering, physics, and astronomy, and is highly useful in many other fields. Some knowledge of higher mathematics is of great use in some of the professions and the study of mathematics is strongly recommended as a training for others, notably law, business, and medicine.

3. You may be in doubt what you will do after college. There is a good chance that you will need a mathematical background for your career. Fundamental courses should be begun early in college. Ignorance of mathematics may hinder your progress later and make you mediocre in your line.

4. The study of mathematics affords abundant opportunity for the exercise of numerous valuable mental traits. Ideals of accuracy, of careful consideration of data, of constantly checking conclusions, and of arranging work in logical order may result from proper instruction in this subject.

5. Mathematical problems present a challenge to our sporting instinct to conquer difficulties. Because you are not interested in high school mathematics, it does not necessarily follow that you will dislike this subject in college. There are many instances to the contrary.

6. *To those who like mathematics and have mathematical ability, this study offers opportunity for a scholarly career of study, teaching, and research.*

7. Mathematical information and power are peculiarly difficult to obtain without an instructor. Many other subjects may be more easily studied and read outside of college.

Very truly yours,

COMMITTEE.

The following program was prepared for the Southeastern Section of the New York State Teachers' Association for its annual meeting in New York City on October 31st and November 1st:

1. Mathematics and Life,
   W. S. Schlauch, High School of Commerce
2. Mathematics, Now and Then,
   Professor David Eugene Smith
3. Round Table Discussion,
   Arranged by Maurice Crosby, Bronxville
   (a) The Content of the Eighth Grade Course in Mathematics,
       W. D. Reeve, Teachers College
   (b) The Crowded Intermediate Algebra Course,
       Cora H. Townsend, New Rochelle
   (c) The Place of Mathematics Clubs in High Schools,
   (d) The Use of Standard Tests in Geometry,
       Vera Sanford, The Lincoln School

ANNUAL MEETING OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

The 1925 meeting of the National Council of Mathematics Teachers will be held at Cincinnati Saturday, February 21st. The Department of Superintendence meets in Cincinnati the following week. All of the mathematics meetings will be held in the Ball Room of the Sinton Hotel. There will be a dinner for all members at the Sinton Hotel at six-thirty. Also there will be a luncheon at twelve o'clock noon for members of the Executive Committee.

As may be observed, the emphasis of each program is upon investigational and experimental phases. The Cincinnati meeting should prove to be a step forward in the teaching of mathematics. There may be minor changes, but the following is substantially correct:

MORNING PROGRAM

Ballroom, Hotel Sinton, Ten O'Clock

(a) The Measured Results of the Wisconsin Supervised Study Program in Mathematics.
Professor Walter W. Hart, University of Wisconsin

(b) The Possibility of Conceptualizing the Processes of Thinking as They Occur in Plane Geometry
Miss Winona Perry, The Lincoln School

(c) Individual Instruction in Ninth Grade Algebra
Mr. C. M. Stokes, New Trier Township High School, Kenilworth, Illinois

(d) Annual Statement by the Editor of the *Mathematics Teacher*
Dr. John R. Clark, The Lincoln School of Teachers College

(e) What Algebra Is Retained by College Freshmen?
Professor Walter Crosby Eells, Whitman College, Walla Walla, Washington

(f) Brief Business Meeting and Appointment of Nominating Committee

AFTERNOON PROGRAM

Ballroom, Sinton Hotel, Two O'Clock

(a) The Psychological Approach to Curriculum Construction in High School Mathematics
Dr. J. Worth Osburn, Director of Educational Measurements, Dept. of Public Instruction, State of Wisconsin

(b) The Problem of Drill in the Seventh and Eighth Grades
Professor O. S. Lutes, State University of Iowa

(c) The Relation of Standard Tests to the Reorganization of Mathematics.
Professor Clifford Brewster Upton, Teachers College, Columbia University